The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme

Applied Mathematics - Level 03

ADU3300/ADE3300 - Vector Algebra

Final Examination - 2024/2025

Duration: - Two Hours

Date: 27. 11. 2024 **Time: 01.30 p.m. - 03.30 p.m.**

General Instructions

- This paper consists of **TWO** sections, Section A and Section B. Section A is **compulsory**, it consists of **ONE** Essay Question.
- Section B consists of FIVE Essay Questions and answer only **THREE** of them.

SECTION A

Answer all questions.

1. a) Given that the vectors $\underline{a}, \underline{b}$, and $\underline{c}$ form a basis, and the vectors:

$$\underline{u} = \underline{a} + 2\underline{c},\ \underline{v} = 2\underline{b} - \underline{c}, \text{ and } \underline{w} = \underline{a} - \underline{b}.$$

Verify that the vectors $\underline{u}$, $\underline{v}$, and $\underline{w}$ are linearly independent.

b) The position vectors of the points P and Q are $2\underline{i} + 4\underline{j} - 3\underline{k}$ and $7\underline{i} - 2\underline{j} + 5\underline{k}$ respectively. Find the angle between $\overrightarrow{PQ}$ and vector $\underline{a} = 2\underline{i} + 2\underline{j} - \underline{k}$.

c) Find the plane through the point $A\ (2, 3, 1)$ and normal to the vector $\underline{i} + 3\underline{j} - 2\underline{k}$.

d) Show that the vector function $\underline{G}(t) = t\underline{i} + (3t\cos t)\underline{j} + (4t\sin t)\underline{k}$ lies on the hyperboloid $x^2 - \frac{y^2}{9} - \frac{z^2}{16} = 0$.

e) Find the Cartesian equation of an ellipse whose center is at the point with position vector $5\underline{i} - 3\underline{j}$ and having major and minor axes of lengths 10 and 6 respectively, which are parallel to the $x-$ and $y-$axes.

SECTION B

Answer THREE questions ONLY.

2. a) Given the vectors $\underline{a} = (2, 3, -1)$, $\underline{b} = (4, -2, 5)$, and $\underline{c} = (-1, 4, 3)$:

   i) Calculate the vector triple product $\underline{a} \times (\underline{b} \times \underline{c})$.

   ii) Hence, verify the vector triple product identity $\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a} \cdot \underline{c})\underline{b} - (\underline{a} \cdot \underline{b})\underline{c}$.

   iii) Find the expression $(\underline{a} \times \underline{d}) \cdot (\underline{b} \times \underline{c})$, given $\underline{d} = (1, 1, -2)$.

   b) Let l_1 and l_2 be two straight lines given by $\underline{r} = \underline{i} + 5\underline{j} + 5\underline{k} + \lambda(2\underline{i} + \underline{j} - \underline{k})$ and $\underline{r} = 2\underline{j} + 12\underline{k} + \mu(3\underline{i} - \underline{j} + 5\underline{k})$ respectively.

   i) Find the intersection point of the two lines.

   ii) The point A lies on l_1 when $\lambda = 1$ and the point B lies on l_2 when $\mu = 1$. Obtain the vector equation of the line l_3 which passes through the points A and B.

3. a) Using the result $\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a} \cdot \underline{c})\underline{b} - (\underline{a} \cdot \underline{b})\underline{c}$, where $\underline{a}$, $\underline{b}$ and $\underline{c}$ are given vectors, show that:

   i) $\underline{i} \times (\underline{a} \times \underline{i}) + \underline{j} \times (\underline{a} \times \underline{j}) + \underline{k} \times (\underline{a} \times \underline{k}) = 2\underline{a}$,

   ii) $(\underline{a} \times \underline{b}) \cdot (\underline{c} \times \underline{d}) = (\underline{b} \cdot \underline{d})(\underline{a} \cdot \underline{c}) - (\underline{b} \cdot \underline{c})(\underline{a} \cdot \underline{d})$,

   iii) Deduce that $(\underline{a} \times \underline{b}) \cdot [(\underline{b} \times \underline{c}) \times (\underline{c} \times \underline{a})] = [\underline{a} \ \underline{b} \ \underline{c}]^2$,

   iv) Using part ii) show that $(\underline{a} \times \underline{b}) \cdot (\underline{c} \times \underline{d}) + (\underline{b} \times \underline{c}) \cdot (\underline{a} \times \underline{d}) + (\underline{c} \times \underline{a}) \cdot (\underline{b} \times \underline{d}) = 0$.

   b) The position vectors of the points P, Q, and R are $\underline{p} = (3, 7, -1)$, $\underline{q} = (1, 2, -3)$, and $\underline{r} = (5, 4, 2)$ respectively. Find the area of the triangle PQR.

4. a) i) Show that the vectors $\underline{a} = (2, 3, -1)$, $\underline{b} = (0, 4, 5)$, and $\underline{c} = (1, 6, 2)$ are non-coplanar.

ii) Find the volume of the parallelepiped formed by these vectors $\underline{a}, \underline{b}$, and $\underline{c}$.

b) Find the cartesian equation of the plane containing the line $\frac{x-2}{5} = \frac{y+3}{-6} = \frac{z-4}{2}$ and which is parallel to the line $\frac{x+1}{7} = \frac{y-2}{5} = \frac{z+3}{-8}$.

5. a) i) A particle moves along a curve, whose parametric equations are

$$x = 2\cos 2t, y = 2\sin 2t, z = \frac{3}{2}e^{-2t}, \text{ where } t \text{ is the time.}$$

Find its velocity at time t, and the speed and acceleration at $t = 0$.

ii) Let $\underline{a}, \underline{b}$ be constant vectors and ω be a constant scalar. Given vector $\underline{r}$ such that,

$$\underline{r}(t) = \cos\omega t\ \underline{a} + \sin\omega t\ \underline{b}, \text{ show that } \underline{r} \times \frac{d\underline{r}}{dt} = \omega\, \underline{a} \times \underline{b}.$$

b) Find the work done on a particle constrained to move from (0,0) to (3,9) along the parabola $y = x^2$ by the field of force $\underline{F} = y\underline{i} + x\underline{j}$.

6. a) Given the parametric equations of a circle:

$$\underline{r}(\theta) = (a + a\cos\theta)\underline{i} + a\sin\theta\, \underline{j}$$

i) Prove that this parametric equation represents a circle of radius a centered at $(a, 0)$.

ii) Determine the point on the circle where $\theta = \frac{\pi}{3}$.

iii) Verify if the point $(2a, \frac{a}{\sqrt{3}})$ lies on the circle.

b) Given the space curve $\underline{r}(t) = \sin t\, \underline{i} + \cos t\, \underline{j} + t\underline{k}$, find :

i) the unit tangent vector,

ii) the principal normal and the curvature of the space curve.

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