THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. Degree Programme

OPEN ELECTIVE- LEVEL 05

ADU5319- DESIGN AND ANALYSIS OF EXPERIMENTS

FINAL EXAMINATION 2024/2025

Duration: Two Hours

Date: 03.05.2025	Time: 01.30p.m- 03.30p.m

Instructions:

- This question paper consists of 06 questions and 06 pages.
- This question paper consists of two parts: Part *A* and Part *B*. **Part *A* is compulsory.** Part *B* consists of five (05) essay-type questions of which **three (03) are to be answered.**
- Statistical tables are attached at the end.
- Non-programmable calculators are permitted.
- Consider the **level of significance as 0.05** for all the tests.

<u>PART *A*</u> *(Question 01 is compulsory)*

Question 01

Identify the following criteria in each of the following experimental situations:

  (i) treatment structure
  (ii) design
  (iii) Treatments/treatment combinations
  (iv) Number of treatments/ treatment combinations
  (v) Experimental units
  (vi) Number of replications
  (vii) Response variable
  (viii) Blocking factors (if any)

a) A researcher is interested in studying the effect of fertilizer type (*Organic, Chemical,* and *Mixed*) and irrigation level (*High, Low*) on the yield of tomato plants. Eighteen similar plots have been randomly assigned to the different combinations of fertilizer types and irrigation levels. After six months, the growth (in cm) of each plant is recorded.

b) A car tyre company intends to test four different types of rubber improvers for their effects on the wear of tyres. Four test cars are available and each improver can be tested by the company's mechanics on one tyre of each car. The cars will be driven under normal conditions for six months, after which the wear of each tyre on each car will be measured.

c) A food scientist wishes to test four different packaging methods (*P1, P2, P3, P4*) to determine their effect on the shelf life of a new snack product. The shelf life may vary depending on the storage room temperature (*Cold, Cool, Warm, Hot*) and the storage shelf position (*Top, Upper-Middle, Lower-Middle, Bottom*). It is assumed that, for a given packaging method, the differences in shelf life between storage temperatures do not depend on shelf position, and the differences due to shelf position are consistent across different temperature conditions. Sixteen snack products are used in this experiment. The shelf life (in days) is recorded for each product.

d) In an experiment to investigate the effect of paper colour (blue, green, orange) on response rates for questionnaires distributed in supermarket parking lots, 15 representative supermarket parking lots were selected, and each colour was randomly assigned to five lots. The response rates were recorded as percentages.

e) A sports shoe manufacturer is testing four different types of insole materials to evaluate their impact on comfort during physical activity. To conduct this study, 20 athletes are recruited, and each athlete tries all four insole types, using one type per day in a random order. After each session. the athletes rate their comfort level on a scale from 1 to 10.

PART *B (Answer 3 questions only)*

Question 02

A distributor of petroleum products. overseeing ten service stations. aimed to determine if there are significant differences among these stations regarding overall quality and effectiveness. To assess this, five professional raters provided ratings for each of the ten stations. The results are presented below:

Rater	Service stations						Total
	1	*2*	*3*	*4*	*5*	*6*	
A	99	90	85	75	70	92	511
B	96	80	88	70	51	91	476
C	95	48	75	71	93	93	475
D	98	70	82	73	94	90	507
E	97	62	80	76	92	89	496
Total	485	350	410	365	400	455	2465

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Total sum of squares = 5316.2

(i) Test whether the mean rating scores for the stations differ significantly at a 5% significance level. Write down your conclusion.

(ii) Test whether the average ratings provided by the five professional raters differ significantly at a 5% significance level. Write down your conclusion.

(iii) Based on the results, what advice would you offer the distributor?

(iv) Construct a 95% confidence interval for the difference between the average rating of stations 1 and 2.

(v) Interpret the results you obtained in part(iv).

Question 03

A researcher is interested in evaluating the effectiveness of four different teaching methods (*A, B, C, and D*) on students' performance. Since student performance can be affected by both the time of day (*Morning, Midday, Afternoon, Evening*) and the teacher (*T1, T2, T3 and T4*), the researcher uses '*X*' Design to control for these two sources of variation. Each teaching method is randomly assigned so that it appears once in each row and once in each column. The students' scores out of 100 after being taught using each method are given below:

Time/Teacher	*T1*	*T2*	*T3*	*T4*	*Total*
Morning	85 (*B*)	88 (C)	84 (D)	90 (A)	347
Midday	82 (C)	80 (D)	89 (A)	86 (B)	337
Afternoon	81 (D)	87 (A)	83 (B)	85 (C)	336
Evening	86 (A)	88 (B)	84 (C)	82 (D)	340
Total	334	343	340	343	1360

Following values are given.

Total sum of squares = 130
Sum of squares of Teacher = 13.5

Teaching Method	A	B	C	D
Total	352	342	339	327

(i) Clearly explaining the notations and stating the assumptions, write down the mathematical model for this '*X*' design.

(ii) Estimate the treatment effects.

(iii) Test whether there is a significant difference among teaching methods at a 5% level of significance and write down your conclusions.

(iv) Estimate the standard error of teaching method *A*.

(v) Test the hypothesis that the effect of teaching method (*A*) and (*B*) is equal.

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Question 04

(i) Explain two instances when the factorial design is preferred over a completely randomized design.

(ii) Write down formulas for computing the main effects, interaction effects and their sums of squares for a 2^2 factorial experiment with 'r' number of replicates, run in a completely randomized design with usual notations.

A baker is investigating how the quality of bread is affected by the type of flour used and the baking time. He selects two types of flour (*all-purpose, whole wheat*) and two baking durations 25, 35) in minutes. For every combination of flour type and baking time, he bakes three loaves of bread. Each loaf is evaluated using a quality index, which is a continuous score representing the bread's quality. The values are given in the following table.

Flour	Baking times (mins)	
	25	35
all-purpose	3.7	4.1
	4.2 (10.8)	4.3 (12.6)
	2.9	4.2
whole wheat	4.1	4.9
	4.5 (13.2)	4.4 (14.2)
	4.6	4.9

Within brackets are cell totals. Grand total = 50.8
Total sum of squares = 3.2267

(iii) How many treatments/ treatment combinations are there in this experiment? What are they?

(iv) Test whether the effects of types of flour and baking time on the quality of bread are significant at a 5% significance level. Clearly explain your answer.

(v) Compute the estimated standard error of treatment effects.

Question 05

(i) Briefly explain the fundamental principles of experimental designs.

(ii) Discuss the limitations of CRD when applied to experiments with high environmental variability.

Fifteen students participated in an experiment to assess the effect of study habits on material retention. Three different types of study habits were tested, and the fifteen students were randomly assigned to one of these types.

Habit 1: control-read the material only
Habit 2: reading the material and then producing a summary
Habit 3: skimming the material, thinking of key questions and then reading the passage properly

Each student was then assessed on their knowledge of the material through multiple-choice questions. Results were as follows:

Study Habit	Scores					Total
Habit 1	22	30	14	28	31	125
Habit 2	32	37	42	28	21	160
Habit 3	44	37	48	35	31	195
						480

Error sum of squares = 652

(iii) State the null and alternative hypotheses to test whether the study methods significantly affect material retention.

(iv) Write down two meaningful contrasts and test whether they are orthogonal.

(v) Test whether the two contrasts mentioned in part(iv) are significant. Write down your conclusions.

(vi) Based on the results in part (v), what advice would you give regarding study habits for material retention?

Question 06

(i) Briefly explain a split-plot design.

A sports scientist is interested in investigating the effects of training intensity (*Low, Medium, High, Very High*) and diet plans (*D1, D2*) on the endurance times (in minutes) of athletes during a treadmill test. The study is conducted at two training centres to account for differences in training facilities. Each centre has four groups of athletes, with each group randomly assigned one of the four training intensity levels. Each group is then divided into two subgroups, where one subgroup follows Diet Plan *D1* and the other follows Diet Plan *D2*. The recorded endurance times are as follows:

Intensity		Centre 1	
		Diet	
		D1	*D2*
	Low	25.1	27.3
	Medium	26.8	28.0
	High	28.3	29.6
	Very High	31.2	32.0

Intensity		Centre 2	
		Diet	
		D1	*D1*
	Low	24.5	26.0
	Medium	25.2	26.7
	High	26.3	27.5
	Very High	29.1	30.6

The following summary statistics were computed by whole plot and sub plot.

Total Sum of Squares (SS Total)	75.22
Total Sum of Squares of whole plot (SS Total wp)	66.82
Total Sum of Squares of sub plot (SS Total sp)	64.68
Sum of Square Centre	9.61

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(ii) Write down the treatment and design structures for the whole plot and sub-plot parts of the experiment.

(iii) Carry out an ANOVA and test the significance of the effects at a 5% level. Write down your conclusions.

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Appendix 4a

5 per cent Points of the *F*-distribution

Column represents degrees of freedom (ν_1) for numerator of *F*-test
Row represents degrees of freedom (ν_2) for denominator of *F*-test

	1	2	3	4	5	6	7	8	9	10	12	24	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	249.1	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.45	19.50
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.785	8.745	8.638	8.526
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.774	5.628
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.678	4.527	4.365
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.841	3.669
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.410	3.230
8	5.318	4.459	4.066	3.838	3.688	3.581	3.500	3.438	3.388	3.347	3.284	3.115	2.928
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	2.900	2.707
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.913	2.737	2.538
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.609	2.405
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.505	2.296
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.420	2.206
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.349	2.131
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.475	2.288	2.066
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.235	2.010
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.190	1.960
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.150	1.917
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.114	1.878
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.082	1.843
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321	2.250	2.054	1.812
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.226	2.028	1.783
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275	2.204	2.005	1.757
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255	2.183	1.984	1.733
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236	2.165	1.964	1.711
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220	2.148	1.946	1.691
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204	2.132	1.930	1.672
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190	2.118	1.915	1.654
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177	2.104	1.901	1.638

Critical values of *t* for two-tailed tests

Significance level (α)

Degrees of freedom (df)	0.2	0.15	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	4.165	6.314	12.706	25.452	63.657	127.321	636.619	1273.239
2	1.886	2.282	2.920	4.303	6.205	9.925	14.089	31.599	44.705
3	1.638	1.924	2.353	3.182	4.177	5.841	7.453	12.924	16.326
4	1.533	1.778	2.132	2.776	3.495	4.604	5.598	8.610	10.306
5	1.476	1.699	2.015	2.571	3.163	4.032	4.773	6.869	7.976
6	1.440	1.650	1.943	(illegible)	2.969	3.707	4.317	5.959	(illegible)
7	1.415	1.617	1.895	(illegible)	2.841	3.499	4.029	5.408	(illegible)
8	1.397	1.592	1.860	(illegible)	2.752	3.355	3.833	5.041	(illegible)
9	1.383	1.574	1.833	(illegible)	2.685	3.250	3.690	4.781	(illegible)
10	1.372	1.559	1.812	(illegible)	2.634	3.169	3.581	4.587	(illegible)
11	1.363	1.548	1.796	(illegible)	2.593	3.106	3.497	4.437	4.883
12	1.356	1.538	1.782	(illegible)	2.560	3.055	3.428	4.318	4.716
13	1.350	1.530	1.771	(illegible)	2.533	3.012	3.372	4.221	4.597
14	1.345	1.523	1.761	(illegible)	2.510	2.977	3.326	4.140	4.498
15	1.341	1.517	1.753	(illegible)	2.490	2.947	3.286	4.073	4.417
16	1.337	1.512	1.746	(illegible)	2.473	2.921	3.252	4.015	(illegible)
17	1.333	1.508	1.740	(illegible)	2.458	2.898	3.222	3.965	(illegible)
18	1.330	1.504	1.734	(illegible)	2.445	2.878	3.197	3.922	(illegible)
19	1.328	1.500	1.729	(illegible)	2.433	2.861	3.174	3.883	(illegible)
20	1.325	1.497	1.725	(illegible)	2.423	2.845	3.153	3.850	(illegible)
21	1.323	1.494	1.721	2.080	2.414	2.831	3.135	3.819	4.110
22	1.321	1.492	1.717	2.074	2.405	2.819	3.119	3.792	4.077
23	1.319	1.489	1.714	2.069	2.398	2.807	3.104	3.768	4.047
24	1.318	1.487	1.711	2.064	2.391	2.797	3.091	3.745	4.021
25	1.316	1.485	1.708	2.060	2.385	2.787	3.078	3.725	3.996
26	1.315	1.483	1.706	2.056	2.379	2.779	3.067	3.707	3.974
27	1.314	1.482	1.703	2.052	2.373	2.771	3.057	3.690	3.954
28	1.313	1.480	1.701	2.048	2.368	2.763	3.047	3.674	3.935
29	1.311	1.479	1.699	2.045	2.364	2.756	3.038	3.659	3.918
30	1.310	1.477	1.697	2.042	2.360	2.750	3.030	3.646	3.902
40	1.303	1.468	1.684	2.021	2.329	2.704	2.971	3.551	3.788
50	1.299	1.462	1.676	2.009	2.311	2.678	2.937	3.496	3.723
60	1.296	1.458	1.671	2.000	2.299	2.660	2.915	3.460	3.681
70	1.294	1.456	1.667	1.994	2.291	2.648	2.899	3.435	3.651
80	1.292	1.453	1.664	1.990	2.284	2.639	2.887	3.416	3.629
100	1.290	1.451	1.660	1.984	2.276	2.626	2.871	3.390	3.598
1000	1.282	1.441	1.646	1.962	2.245	2.581	2.813	3.300	3.492
Infinite	1.282	1.440	1.645	1.960	2.241	2.576	2.807	3.291	3.481