The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
Final Examination 2024/2025
Level 04 Applied Mathematics
ADU4300/ADE4300– Statistical Distribution Theory

Duration: - Two hours

Date: - 09-12-2024 **Time: - 1.30 p.m. to 3.30 p.m.**

Non programmable calculators are permitted.
Answer four questions only.

1.

A company that produces a certain electrical product claims that the life time X (in Years) has the density function

$$f(x) = \frac{1}{10} e^{-[\frac{x}{10}]} \;\; ; \;\; x \geq 0.$$

(i) Find the expected lifetime of a randomly selected product.

(ii) Find the cumulative distribution function of a randomly selected product.

(iii) Find the probability that a randomly selected product will not fail within 8 months.

(iv) Find the probability that a randomly selected product will fail within 8 months to 12 months.

(v) Find the probability that a randomly selected product will fail before 10 months or after 15 months.

(vi) Find the highest lifetime of the lowest 50% lifetimes of the product.

2.

A team of 3 is chosen at random from 5 girls and 3 boys. Let X be the number of girls in the team.

(i) Find the probability mass function of X.

(ii) Find the probability that the team chosen has more girls than boys.

(iii) Find the expected number of boys in the team and the variance of the number of boys in the team.

(iv) Suppose the probability of giving a party by boys when all boys are selected to the team is 0.6 and probability of giving a party by girls when more girls than boys are selected to the team is 0.8. No party will be given by girls or boys if only two boys are selected to the team. Let Z be the event of giving a party after the selection of the team

a) Find the probability of having a party after the selection of the team.

b) Find the joint probability mass function of X and Z.

3.

(a) Proportion p of the pins in a box are out of the specifications. A random sample of n pins was drawn with replacement. Suppose X of them were out of the specifications. The probability mass function of X is given by, $P_X(X = x) = n_{C_x} p^x (1-p)^{n-x}; \quad x = 0,1,2,3, \dots \dots \dots n$

Let $M_x(t)$ be the moment generating function of X.

(i) Show that $M_x(t) = [1 + p(e^t - 1)]^n$.

(ii) Using part (i), show that $E(X) = np$ and $Var(X) = np(1-p)$.

(b) Batch that consists of 200 coil springs from a production process, is checked for conformance to customer requirements. From the past experience, the mean number of nonconforming coil springs in a batch of 200 is 20. According to the past experience, what is the probability that tested batch consists of more than 25 nonconforming coil springs?

(c) A restaurant kitchen has two food mixing machines A and B. The average number of times A breaks down per week is 0.4 and the average number of times B breaks down per week is 0.1. Find the probability that total number of breakdowns for a period of four weeks exceeds 4. You may assume that breakdowns are immediately repaired and put back to work.

4.

Life time of a light bulb manufactured by ***ABC*** Company is normally distributed with mean 600 days and a standard deviation of 60 days.

(i) Find the probability that life time of a randomly selected light exceeds two years.

(ii) The production manager of the ***ABC*** Company plans to set a warranty period (in days) such that 95% of the bulbs should not fail during the warranty period. Calculate the warranty period.

(iii) Random sample of 10 bulbs from the above population were tested and sample mean $\bar{X}$ was estimated. Find the probability that sample mean $\bar{X}$ exceeds 525 days.

(iv) From past experience it is known that monthly mean salary of employees in ***ABC*** Company is Rs. 50000 with standard deviation of Rs. 20000. Suppose a sample of 100 employees were selected for a survey. Find the probability that mean salary of the selected sample of employees will be less than Rs.53000. Clearly state any assumptions or theorems used in the answering to this part.

5.

Suppose that $X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9$ are independent random variables described as

$X_1 \sim N(3,4)$ $X_2 \sim N(2,9)$ $X_3 \sim exp(3)$ $X_4 \sim gamma(3,3)$ $X_5 \sim exp(3)$ $X_6 \sim Bin(6,0.4)$

$X_7 \sim Bin(4,0.4)$ $X_8 \sim poison(2)$ $X_9 \sim poison(3)$

Find the following probabilities. Show your calculations and state the justifications clearly.

(i) $\Pr\left[\left(\frac{X_2-2}{3}\right)^2 < 5.024\ \right]$

(ii) $\Pr\left[X_3 + X_5 + X_4\right) < 1\ )$

(iii) $\Pr\left[\ 18 < (X_1 + 3X_2) < 25.62\ \right]$

(iv) $\Pr[(X_6 + X_7) > 6\ )]$

(v) $\Pr[(2X_8 + X_9) > 5\ )]$

6.

A certain shop sells two brand of VCR *A* and *B*. Let *X* denote the number of brand *A* VCR machines sold per day and Y denote the number of brand *B* VCR machines sold per day. The following table shows the joint probabilities, according to the past data.

P(x,y)		*x*		
		0	1	2
y	0	0.10	0.04	0.02
	1	0.30	0.14	0.06
	2	0.08	0.2	*k*

(i) Find the value of *k*.

(ii) Find the marginal distribution function of ***X***.

(iii) What is the expected total number of sales of VCR on a randomly selected day?

(iv) What is the probability that on a randomly selected day, the number of brand ***B*** VCR machines sold is more than that of brand *A*?

(v) On a particular day salesman of the shop has sold their first brand ***B*** VCR at 10.00 a.m. Assume that the shop opens at 9.00 a.m. and closes at 5.00 p.m. What is the probability of no sales of brand ***A*** VCR on that day?

(vi) Find the conditional probability mass function of brand ***A*** VCR machines sold on a randomly selected day given that at least one brand ***B*** VCR machines is sold on that day.

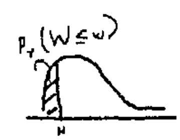
(vii) Are the sales of brand *A* and brand ***B*** independent? Justify your answer.

Table of $\chi^2_{\alpha,\nu}$ quantiles (χ^2 table)

df	α							
ν	0.99	0.975	0.95	0.90	0.1	0.05	0.025	0.01
1	0	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.02	0.051	0.103	0.211	4.605	5.991	7.378	9.21
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.61	9.236	11.07	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	11.449	16.812

Let $X \sim \chi^2_\nu$ and α be a probability. This table contains the upper α quantiles $\chi^2_{\alpha,\nu}$ of the χ^2_ν distributions such that $\Pr(X > \chi^2_{\alpha,\nu}) = \alpha$

Left tail values of Standard Gamma Table
W - gamma(α,1)
This table contain the probabilities Pr(W ≤ w)



w	α					
	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432

Table of Standard Normal Probabilities

Let $Z \sim N(0,1)$. This table contains the probabilities $Pr(z \leq Z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010