00206

The Open University of Sri Lanka

B.Sc/B.Ed. Degree Programme – Level 04

Final Examination – 2024/2025

Pure Mathematics

PEU4300 – Real Analysis 1

Duration: - Two Hours.

Date: - 25.11.2024 **Time: - 01.30 p.m.-03.30 p.m.**

Answer Four Questions only.

(01) (a) Using the definition of limit, Prove that

$$\lim_{n\to\infty} \frac{n^3+3n^2+5n+2}{2n^3+n+1} = \frac{1}{2}.$$

(b) Let $\langle x_n \rangle$ be the sequence given by $x_n = \frac{n-8}{n+2}$ for each $n \in \mathbb{N}$.

Prove that $\langle x_n \rangle$ is increasing and bounded.

(c) Prove or disprove that $\left|\frac{3n+1}{2n+1} - \frac{1}{2}\right| < \frac{4}{5}$ for every $n \in \mathbb{N}$.

(02) Let $x_1 > 0$. Define the rest of the sequence $\langle x_n \rangle$ by $x_{n+1} = \frac{1}{2}\left(x_n + \frac{4}{x_n}\right)$

For each $n \in \mathbb{N}$. Prove that,

(i) $x_n > 0$ for each $n \in \mathbb{N}$,

(ii) $\langle x_n \rangle$ is bounded below,

(iii) $\langle x_n \rangle$ is monotonically decreasing,

(iv) $\langle x_n \rangle$ converges and $\lim_{n\to\infty} x_n = 2$.

(03) (a) Using the definition of a sequence diverges to infinity, prove that the sequence

$\left\langle\frac{2n\sqrt{n}+1}{\sqrt{n}+2}\right\rangle$ diverges to ∞.

(b) write down the definition of a Cauchy sequence.

Prove that $\left\langle\frac{n^2-5}{n^2+2}\right\rangle$ is Cauchy.

(c) Prove that $\left\langle(-1)^n\left(\frac{n+1}{n}\right)\right\rangle$ is not a Cauchy sequence.

(04) (a) Let $\langle x_n\rangle$ be a convergent sequence such that $\lim_{n\to\infty} x_n = x$. Prove that

$\sum_{n=1}^{\infty}(x_n - x_{n+1})$ converges. Find $\sum_{n=1}^{\infty}(x_n - x_{n+1})$.

(b) Show that $\sum_{n=1}^{\infty}\frac{1}{(n+2)(n+4)} = \frac{7}{24}$.

(c) Determine whether each of the following geometric series is convergent or divergent.

(i) $\sum_{n=1}^{\infty}\frac{(-4)^{3n}}{5^{n-1}}$ (ii) $\sum_{n=1}^{\infty} 9^{-n+2}\, 4^{n+1}$.

(05) Determine the convergence or divergence of each of the following series:

(i) $\sum_{n=1}^{\infty}\frac{|\cos n|}{n^{\frac{3}{2}}}$, (ii) $\sum_{n=1}^{\infty}\frac{n!}{2^{2n-1}}$,

(iii) $\sum_{n=1}^{\infty}\frac{1}{3n^2-2n+7}$, (iv) $\sum_{n=1}^{\infty}\left(\sqrt[n]{n}-1\right)^n$,

(v) $\sum_{n=1}^{\infty}\frac{5^n}{2^n+5}$.

(06) (a) Determine the radius of convergence of each of the following power series:

(i) $\sum_{n=1}^{\infty} \frac{(n!)^3}{3n!} x^n$,

(ii) $\sum_{n=1}^{\infty} \frac{1}{n^3 3^n} x^n$.

(b) Find whether each of the following series is conditionally convergent, absolutely Convergent or divergent:

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+2)}$,

(ii) $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\log n}$,

(iii) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+1}{n+2}\right)$.