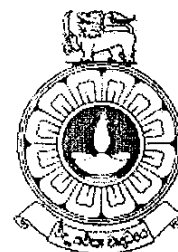


The Open University of Sri Lanka  
 B.Sc./B. Ed Degree Programme  
 Final Examination – 2024/2025  
 Pure Mathematics – Level 04  
 PEU4315 – Continuous Functions  
 Duration: Two hours.



Date: 11.12.2024

Time: 09.30 a.m. – 11.30 a.m.

Answer **Four** Questions only.

1. (a). State the definition of a limit point.  
 (b). Let  $f(x) = x^3$ , for all  $x \in \mathbb{R}$ .  
 Prove that  $\lim_{x \rightarrow 2} f(x) \neq 4$ .  
 (c). Prove that the set of natural numbers,  $\mathbb{N}$  has no limit points.
2. (a). Suppose  $E \subseteq \mathbb{R}$ ,  $c$  is a limit point of  $E$ ,  $f, g: E \rightarrow \mathbb{R}$  are functions such that  $\lim_{x \rightarrow c} f(x)$  and  $\lim_{x \rightarrow c} g(x)$  exist. Prove that  $\lim_{x \rightarrow c} f(x)g(x)$  exists and  $\lim_{x \rightarrow c} f(x)g(x) = \lim_{x \rightarrow c} f(x) \lim_{x \rightarrow c} g(x)$ .  
 (b). Let  $E = \mathbb{Q} \cap [0, 2]$  and  $f: E \rightarrow \mathbb{R}$  be  $f(x) = (x^2 + 3)\sqrt{x + 7}$ ,  $x \in E$ . Find  $\lim_{x \rightarrow 2} f(x)$ .
3. (a). By using the Sandwich theorem, prove that  $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ .  
 (b). Prove that if  $\lim_{x \rightarrow \infty} f(x) = l$  exists for some  $l \in \mathbb{R}$ , then  $l$  is unique.  
 (c). Let  $f(x) = \frac{1}{x}$ , for each  $x \in (0, \infty)$ . Prove that  $f$  is unbounded and  $\lim_{x \rightarrow \infty} f(x) = 0$ .
4. (a). Let  $f, g$  be functions,  $c_1, c_2$  be real numbers such that  $(c_1, \infty) \subseteq \text{Domn}(f)$  and  $(c_2, \infty) \subseteq \text{Domn}(g)$ . Suppose  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow \infty} g(x)$  exist. Prove that  $\lim_{x \rightarrow \infty} [f(x)g(x)]$  exists and  $\lim_{x \rightarrow \infty} [f(x)g(x)] = \lim_{x \rightarrow \infty} f(x) \lim_{x \rightarrow \infty} g(x)$ .  
 (b). Let  $f(x) = \frac{1}{(x-3)^2}$ ,  $x \in \mathbb{R} \setminus \{3\}$ . Show that  $\lim_{x \rightarrow 3} f(x) = \infty$ .

5. (a). (i). Suppose  $f$  is a function defined on an interval  $(a, b)$  and  $f$  is continuous at  $c$ , where  $c \in (a, b)$ . Prove that  $|f(x)|$  is continuous at  $c$ .
- (ii). Is it true that  $|f(x)|$  is always discontinuous at a point where the function  $f(x)$  is discontinuous at the same point? Justify your answer.
- (b). Let  $f(x) = \frac{x+1}{x^2+2}, x \in \mathbb{R}$ .
- Using definition, prove that  $f(x)$  is continuous at 3.
6. (a). Define  $h: (-1, 5) \rightarrow \mathbb{R}$  by  $h(x) = \sqrt{5 - x^2 + 4x}$  for each  $x \in (-1, 5)$ . Show that  $h$  is continuous on  $(-1, 5)$ .
- (b). Suppose  $f$  is a function defined on an interval  $I$  and  $f$  is uniformly continuous on  $I$ . Prove that  $f$  is continuous on  $I$ .

...End...