THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./ B.Ed. Degree Programme
APPLIED MATHEMATICS - LEVEL 03
ADU3302 - Differential Equations
Final Examination - 2024/2025
DURATION: TWO HOUR

Date: 20.05.2025 **Time: 1.30 p.m. to 3.30 p.m.**

General Instructions

- This paper consists of **Two** sections, Section **A** and Section **B**. Section **A** is **compulsory** and it consists of **FIVE** Structured Essay Questions and carries 100 marks.
- Section B consists of **FIVE** essay-type questions and answer only **THREE** of them. Each question in Section B carries 100 marks.
- This paper consists of 03 pages.

SECTION A

1. Answer all the questions in this section.

(i) Consider the following differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \cos(x)$$

(a) Find the order of the differential equation.
(b) Find the degree of the differential equation.
(c) Determine whether the equation is linear or nonlinear.

(ii) Use an integrating factor to reduce the following differential equation to an exact differential equation

$$(x^2 + y^2 + 2x)dx + 2ydy = 0.$$

(iii) A hot object is placed in a room with a constant ambient temperature of T_0. According to Newton's Law of Cooling, the rate at which the temperature $T(t)$ of the object changes is proportional to the difference between the object's temperature and the ambient temperature. Develop a differential equation to model this cooling process using proportionality constant k.

(iv) Simplify the following expression

$$[D^2 - 4D + 3]e^{kx}.$$

(v) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$.

SECTION B

Answer **THREE** Questions **ONLY** from this section.

2. (i) Solve the following differential equation using the method of separation of variable

$$\frac{dy}{dx} = (x^2 + 6)(y - 7).$$

(ii) Consider the differential equation $\dfrac{dy}{dx} = \dfrac{x + 2y - 2}{2x - y + 1}$.

(a) Convert the above differential equation into the form $\dfrac{dY}{dX} = \dfrac{X + 2Y}{2X - Y}$ using the substitutions $x = X + h$ and $y = Y + k$, where h and k are to be determined.

(b) Hence, solve the given differential equation.

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3. The number of reported cases of an infectious disease, denoted by x (in thousands), is decreasing over time, where t represents the number of months since the disease was first reported. The rate at which the number of cases is declining is proportional to the square of the number of reported cases. It is assumed that x can be modeled as a continuous variable.

(i) Write down the differential equation in terms of x, t and a proportionality constant k.

(ii) Initially, there were 2,500 reported cases. After one month, the number of cases had decreased to 1,600.

(a) Solve the differential equation to show that

$$x = \frac{40}{9t + 16}.$$

(b) Determine the number of months that will pass until the number of reported cases reaches 250.

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4. (i) Consider the following differential equation:

$$(y^2 \cos x + 2x \cos y)dx + (2y \sin x - x^2 \sin y)dy = 0.$$

(a) Show that the above differential equation is an exact differential equation.

(b) Solve the above differential equation using suitable method.

(ii) Consider the following linear differential equation with a condition.

$$\frac{dy}{dx} + \frac{y}{x} = x^3, \quad y(1) = 2$$

Show that the solution of the above differential equation is:

$$xy = \frac{1}{5}(x^5 + 9).$$

5. Consider the nonhomogeneous ordinary differential equation,

$$D^2y + 4y = 4x^2 + 10e^{-x}.$$

(a) Find a general solution for the associated homogeneous equation.

(b) Find a particular solution of the given nonhomogeneous equation.

(c) Hence write down the general solution of the above non homogeneous differential equation.

(d) Find the solution of the nonhomogeneous equation if $y(0) = 2$ and $y'(0) = 0$.

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6. (i) Consider the second order nonhomogeneous ordinary differential equation,

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x^2 + sinx.$$

(a) Identify the UC-functions and generate the corresponding UC-sets.

(b) Hence, find the particular solution of the above differential equation using the method of Undetermined Coefficients.

(ii) Find all singular points of the following differential equation

$$x(x+3)y'' + x^2y' - y = 0.$$

Then, check whether these points are regular singular points or irregular singular points.

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