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The Open University of Sri Lanka
B.Sc. Degree Programme: Level 05
Department of Computer Science
CSU 5304- Mathematics for Computing
Final Examination-2024/25
Duration: Two Hours Only (2 Hours)

Date: 22.11.2024 **Time: 9.30 am-11.30am**

Answer Four Questions only.

QUESTION 1

i. Give the definition of a function in the context of mathematics.

ii. $f(x)$ is a function defined by;

$$f(x) = \begin{cases} 2x + 4 & x \leq 2 \\ 2x - 1 & x > 2 \end{cases}$$

Find the values of $f(6)$, $f(2)$ and $f(0)$.

iii. p(x) and q(x) are two functions given by $p(x)= \frac{2x}{5} + 7$ and $q(x)= 10x^2-15$; $\forall x \in \mathbb{R}$. Find the composite function $p \circ q(x)$ and give your answer in the form of cx^2+d, where c and d are integers.

iv. When composing a composite function, with only two functions, what is the important condition that should be satisfied by the first function?

(25 marks)

QUESTION 2

i. Write down the conditions to be satisfied in order to perform the following matrix operations.
   a) Addition of two matrices
   b) Multiplication of two matrices
   c) Finding the determinant of a matrix.

ii. The two matrices A and B are defined by;

$$A=\begin{bmatrix} -2 & 1 & 2 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix} 1 & 1 & 2 \\ k+3 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{where } k \in \mathbb{R}.$$

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a) Obtain the matrix $2A^T - B$, where A^T is the transpose of A.

b) (i) Obtain an expression for the determinant of B and simplify it.

(ii) Find the value of k such that B^{-1} does not exist.

iii. For the matrix $C = \begin{bmatrix} 2 & 2\alpha - \beta & -1 \\ 3\alpha - 2\beta & 4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$,

Obtain the values of α and β such that, $C^T = \begin{bmatrix} 2 & -5 & -1 \\ -1 & 4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$ where C^T is the transpose of matrix C.

iv.

a) Consider any two square matrices, write the commutative law for matrices under addition. **(Hint: You can consider the two matrices as *P* and *Q*).**

b) Show that if A is any square matrix, then $(A+I)$ and $(A-I)$ commute, where I is the identity matrix with same order as A.

(25 marks)

QUESTION 3

i. What are the main steps you would follow in Mathematical Induction?

ii. Use Mathematical Induction to prove that for all $n \geq 1$

$$1+4+7+\ldots+(3n-2) = \frac{n(3n-1)}{2}$$

iii. Using Mathematical Induction, to show that $(6^n - 1)$ is divisible by 5 for all $n \in \mathbb{N}$.

(25 marks)

QUESTION 4

i. Define the two terms **"Statement"** and **"Proposition".**

ii. Which of the following sentences are propositions? Find the truth values for the propositions you mentioned in your answers.

a) 3+5=8

b) x+4 >10 ; for every real number x.

iii. State the **Inverse** and the **Converse** of the following implications.

a) We will come for the day school, if it is sunny.

b) If it rains tomorrow, I will stay at home.

iv. Give symbols for the following sets.

a) Set of positive integers.

b) Set of real numbers.

c) Set of complex numbers.

(25 marks)

QUESTION 5

i. Why do we use sets in mathematics?

ii. Define the following terms with regard to sets.

a) Set and Subset

b) Power set

c) Equal set

iii. List the elements of the following sets, where P= {1, 2, 3, ……., 30}.

a) $A=\{x: x \in P, 3<x<15\}$

b) $B=\{x: x \in P, x \text{ is even and } x<20\}$

c) $C=\{ x: x \in P, 4+x=14\}$

d) $D=\{ x: x \in P, x \text{ is a multiple of } 5\}$

iv. A small college of 140 students, require its students to take at least one mathematics course and at least one science course. 60 students study mathematics, 45 students study science and 20 students study both mathematics and science. Use a Venn diagram or any other method in set theory to find the number of students who had satisfied:

a) Exactly one of two requirements

b) At least one of the requirements

c) Neither requirement

(25 marks)

QUESTION 6

i.

a) An arithmetic progression is a sequence of the form a, a+d, a+2d, ……., a+(n-1)d, where "a" is the initial term and "d" is the common difference. Further, "a" and "d" are real numbers. Write a formula for the sum of the first n terms of an arithmetic progression using "a" and "d".

b) In an arithmetic progression, the sum of the first 10 terms is 400 and the sum of the next 10 terms is 1000. Find the common difference and the first term.

ii.

a) A geometric progression is a sequence of the form $a, ar, ar^2, ar^3, \ldots, ar^n$, where a is the initial term and r the common ratio. Write a formula for the sum of the first n terms and hence deduce the sum to infinity.

b) A geometric progression has first term "a", common ratio "r" and sum to infinity is 6. Another geometric progression has first term "2a", common ratio "r^2" and sum to infinity is 7. Find the values of a and r.

iii.

a) Give the definition of "greatest common divisor" (gcd) and "least common multiple" (lcm).

b) Use the definitions of part (iii) (a) and find
   - The gcd of 12 and 30
   - The lcm of 24 and 36

   (All working steps must be given)

iv.

a) Let "b" be an integer and "m" be a positive integer. What is denoted by **b mod m**?

b) Find the value of 13 mod 5.

(25 marks)