The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Pure Mathematics - Level 05
PEU5302 Combinatorics
Final Examination 2024/2025
Duration: Two hours

Date: 01.12.2024   Time: 1:30 p.m. - 3:30 p.m.

INSTRUCTIONS TO CANDIDATES

- There are **SIX** questions in this paper. Answer **FOUR** of them.

- Calculators are **NOT** allowed.

1. There are 9 lecturers and 10 students in the mathematics department. Out of them a committee of 10 members for the society has to be formed. The seating in the committee meeting is to be arranged in a circular arrangement. How many ways are there to arrange the seating in the following cases?

   (a) Choose 5 lecturers and 5 students for the committee, arrange the seating where the lecturers and students alternate.

   (b) Choose 4 lecturers and 6 students for the committee, appoint a lecturer as the president and a student as the secretary, arrange the seating where president the secretary sit together.

   Justify your answers.

   (100 marks)

2. (a) 12 apples are distributed among Amal, Babu and Chathuri. How many ways are there to distribute so that each one gets at least 2 apples? Justify your answer.

   (b) Suppose you throw darts at a target of the shape of a square with the length of a side $1m$. You throw datrs of two colours and 51 of them hit the target. Show that it is always possible to find a square with the length of a side $\frac{1}{5}m$ in which it is possible to find two darts with the same colour. (Hint: Use the pigeonhole principle twice.)

   (100 marks)

3. (a) Show by using the principle of mathematical induction that $2^n \geq n^2$ for any integer n such that $n \geq 4$. (Hint: You may use the result without proof that $n^2 \geq 2n + 1$ when $n \geq 4$)

   (b) Draw a flowchart to find the factorial of a given integer greater than or equal to 1.

   (100 marks)

4. A start-up company has 20 employees which consists of 3 groups A, B and C. The group composition is as follows.

Group	Male	Female
A	2	3
B	5	3
C	3	4

If a committee of 9 is formed at random,

(a) What is the probability that there are exactly 2 males from group B and at least 1 female from group A?

(b) What is the probability that there are at least 1 male from group A and exactly 2 females from group C, given that there are no males in the committee from group B?

(100 marks)

5. Let $n \in \mathbb{N}$ and $r \in \{0, 1, \cdots, n\}$.

(a) Using the binomial expansion of $(1 + x)^{2n}$, show that

$$^{2n}C_r = \sum_{k=0}^{r} {}^nC_k \cdot {}^nC_{r-k}.$$

Hence deduce that

$$^{2n}C_2 = 2 \cdot {}^nC_2 + n^2.$$

(b) Verify the identity $^{2n}C_2 = 2 \cdot {}^nC_2 + n^2$ using a combinatorial argument.

(100 marks)

6. (a) Find the largest term in the binomial expansion of $(2 + 3x)^8$ where $x = \frac{1}{2}$.

(b) Find the constant term in the multinomial expansion of $\left(1 + 3x^2 + \dfrac{4}{x^5}\right)^7$.

(100 marks)

The End.