The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc./ B.Ed. Degree Programme

Department	**: Mathematics**
Level	**: 04**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Newtonian Mechanics I – ADU4301**
Academic Year	**: 2023/24**
Date	**: 22.03.2024**
Time	**: 2.00 p.m. To 4.00. p.m.**
Duration	**: Two Hours.**

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **(6)** questions in **(3)** pages.
3. Answer any **(4)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

1. A particle of unit mass is projected vertically upwards with velocity V in a medium for which the resistance is kv per unit mass when the speed of the particle is v.

(a) Let x be the distance travelled at time t, then show that

$$x = \frac{1}{k}(V - v) + \frac{g}{k^2}\ln\left|\frac{g+kv}{g+kV}\right|.$$

Hence, show that the maximum height H attained by the particle from the point of projection is given by $H = \frac{V}{k} + \frac{g}{k^2}\ln\left|\frac{g}{g+kV}\right|$.

(b) If the particle returns to the point of projection, with speed U after time T then show that

(i) $U + V = \frac{g}{k}\ln\left|\frac{g+kV}{g-kU}\right|$ and

(ii) $U + V = gT$.

2. (a) With the usual notation, show that in plane polar coordinates (r, θ), the velocity $\underset{\sim}{v}$ and acceleration $\underset{\sim}{a}$ of a particle moving in a plane are given by $\underset{\sim}{v} = \dot{r}\underset{\sim}{e}_r + r\dot{\theta}\underset{\sim}{e}_\theta$ and

$\underset{\sim}{a} = (\ddot{r} - r\dot{\theta}^2)\underset{\sim}{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underset{\sim}{e}_\theta$ respectively, where $\underset{\sim}{e}_r$, $\underset{\sim}{e}_\theta$ have the usual meanings.

(b) A particle, of mass m, is held on a smooth table. A string attached to this particle passes through a hole in the table and connects to a particle of mass $3m$. Motion is started by a particle on the table being projected with velocity V at right angles to the string. If a is the original length of the string on the table, show that when hanging particle has descended a distance $a/2$ (assume to be possible) its velocity will be $\frac{\sqrt{3}}{2}\sqrt{ga - V^2}$.

3. (a) Establish the formula $\underset{\sim}{F}(t) = m(t)\frac{d\underset{\sim}{v}}{dt} - \underset{\sim}{u}\frac{dm}{dt}$ for the motion of a particle of varying mas $m(t)$ moving with velocity $\underset{\sim}{v}$ under a force $\underset{\sim}{F}(t)$, the matter being added at a rate $\frac{dm}{dt}$ with velocity $\underset{\sim}{u}$ relative to the particle.

(b) A particle, of mass M, is projected vertically upwards in a cloud. During the motion, the particle absorbs moisture from the stationary cloud so that the particle is at distance x above the point of projection, moving with speed v, it has mass $M(1 + \alpha x)$ where α is a constant. The initial speed of the particle is $\sqrt{2gk}$.

(i) Show that $\frac{dv^2}{dx} + \frac{2\alpha}{1+\alpha x}v^2 = -2g$.

(ii) Show that the greatest height, h is given by $(1 + \alpha h)^3 = 1 + 3k\alpha$.

4. (a) With the usual notation show that the equation of the central orbit of a particle moving in a plane is given by

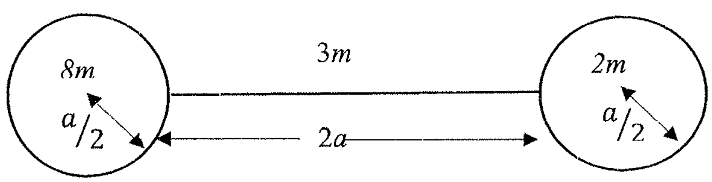
$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2} \quad \text{and} \quad \dot{\theta} = hu^2.$$

(b) A particle is projected from an apse at a distance a form an origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a and moves with central acceleration μ/r^3. Show that the path is $r\cos(\theta/\sqrt{2}) = a$.

5. (a) Show that for a rigid body the change in angular momentum is equal to the angular impulse.

(b) A uniform rod AB of length $2a$ and mass $3m$ is at rest on a smooth, horizontal table and the rod is free to rotate on the table about a smooth, vertical axis through end A. A particle P of mass $\frac{3m}{5}$ moving at u on the table at right angles to AB strikes the rod at C where $AC = \frac{5a}{4}$. P adheres to the rod. Calculate the angular speed with which the rod begins to rotate.

6.



A model of a timing device in a clock consists of a uniform rod, of mass $3m$ and length $2a$, the ends of which are attached two uniform solid spheres, each of radius $\frac{a}{2}$ as shown in the figure. One sphere has mass $8m$ and the other has mass $2m$. The device rotates freely in a vertical plane abut a horizontal axis through the centre of the rod and perpendicular to it.

(a) Show that the moment of inertia of the system about this axis is $\frac{49}{2}ma^2$.

(b) Find the period of small oscillations of the system about its position of stable equilibrium.

(You may assume that the moment of inertia of a uniform solid sphere of radius a and mass m, about an axis passing through the centre is $\frac{2}{5}ma^2$.