The Open University of Sri Lanka
B.Sc. /B.Ed. Degree Programme
Final Examination – 2024/2025
Applied Mathematics – Level 04
ADU4303 – Applied Linear Algebra and Differential Equations

DURATION: TWO HOURS.

Date: 21.05.2025 **Time: 9.30am-11.30am**

Important: Answer FOUR questions only. Answer maximum ONE question from part A and minimum THREE questions from part B.

Part A

1. (a) Prove that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0.$

(b) Discuss the consistency of the following system of simultaneous equations:

$$x - 2y + 3z = 6$$
$$3x + y - 4z = -7$$
$$5x - 3y + \alpha z = \beta.$$

Obtain the complete solution for the consistent cases.

(c) If $A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{pmatrix}$ then determine the values of b such that the rank of A is 3.

2. (a) Transform the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 + 2x_1x_3 - 2x_2x_3 - 2x_1x_2$ to canonical form using an orthogonal transformation.

   (b) Find the eigen values and eigen vectors of the following matrix:

$$A = \begin{pmatrix} 3 & 2+i \\ 2-i & 7 \end{pmatrix}.$$

Part B

1. Find the general solution of each of the systems of simultaneous differential equations, given below in the standard notation:

   (a)
$$\begin{aligned} \dot{x}_1 &= -x_1 + 7x_2 - x_3 \\ \dot{x}_2 &= x_2 \\ \dot{x}_3 &= 15x_2 - 2x_3. \end{aligned}$$

   (b)
$$\begin{aligned} \dot{x}_1 &= 2x_1 + 3x_2 + e^{2t} \\ \dot{x}_2 &= 2x_1 + x_2 + 4e^{2t} \end{aligned}$$

   (c)
$$\begin{aligned} \ddot{x} &= -x + 3y \\ \ddot{y} &= x + y. \end{aligned}$$

2. (a) Find a sinusoidal particular solution for the following system of partial differential equations.

$$\begin{aligned} \ddot{x}_1 &= 8x_1 - 5x_2 + \sin 2t \\ \ddot{x}_2 &= 10x_1 - 7x_2 + 2\cos 2t. \end{aligned}$$

(b) Find the general solution of the following simultaneous partial differential Equations:

$$\frac{\partial u}{\partial x} = 4xy + a\cos ax, \quad \frac{\partial u}{\partial y} = 2x^2 + 9e^{3y}.$$

(c) Find the general solution of the following partial differential Equation:

$$y\frac{\partial u}{\partial y} + 2xy^2 u = y^2.$$

3. (a) Find the equations of the characteristic curves for the partial differential equation

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = 2.$$

and define new variables that could be used to simplify the equation.

(b) Hence obtain the general solution $u(x, y)$ of the partial differential equation given in part (a).

(c) The partial differential equation $\frac{\partial u}{\partial x} - 3\frac{\partial u}{\partial y} = u$ has the solution $u(x, y) = e^x f(y + 3x)$.

If $u(x, y) = y$ on $x = 0$, what is the solution in this case?

4. (a) Consider the partial differential equation

$$y^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2} - \frac{y^2}{x}\frac{\partial u}{\partial x} - \frac{x^2}{y}\frac{\partial u}{\partial y} = 0, \qquad (x \neq 0, \ y \neq 0).$$

Use the transformation $\zeta = x^2 + y^2$ and $\phi = y$ to show that it can be reduced to the standard form $\frac{\partial^2 u}{\partial \phi^2} - \frac{1}{\phi}\frac{\partial u}{\partial \phi} = 0.$

(b) Solve each of the following partial differential equations:

(i) $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0.$

(ii) $2\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} - 6\frac{\partial^2 u}{\partial y^2} = 0.$