00208

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc./ B.Ed. Degree Programme

Department	: Mathematics
Level	: 04
Name of the Examination	: Final Examination
Course Title and - Code	: Newtonian Mechanics I – ADU4301
Academic Year	: 2024/25
Date	: 09.05.2025
Time	: 9.30 a.m. To 11.30. a.m.
Duration	: Two Hours.

1. Read all instructions carefully before answering the questions.
2. This question paper consists of (6) questions in (3) pages.
3. Answer any (4) questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary.
6. Involvement in any activity that is considered as an exam offence will lead to punishment.
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script.

1. A particle is projected with velocity u at an angle α to the horizontal in a medium whose resistance per unit mass is kv where v is the speed of the particle. If R is the range on the horizontal plane through the point of projection, and T is the time of flight, prove that

$$kR\left(k\tan\alpha + \frac{g}{u}\sec\alpha\right) + g\ln\left(1 - \frac{kR}{u}\sec\alpha\right) = 0 \text{ and } T = \frac{1}{k}\ln\left(\frac{u}{u-kR\sec\alpha}\right).$$

2. (a) With the usual notation, show that in plane polar coordinates (r,θ), the velocity $\underline{v}$ and acceleration $\underline{a}$ of a particle moving in a plane are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$ respectively, where $\underline{e}_r$, $\underline{e}_\theta$ have the usual meanings.

(b). A particle, P moves round the circle whose polar equation is $r = 2a\cos\theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, where a is a positive constant. It moves so that its acceleration has no transverse component. Show that

(i) $\dot{r}$ is proportional to $\frac{\sin\theta}{r^2}$.

(ii) the radial acceleration is proportional to $\frac{1}{r^5}$.

3. (a) With the usual notation, show that in intrinsic coordinates, the velocity and acceleration $\underline{a}$ of a particle moving in a plane curve are given by $\underline{v} = \dot{s}\underline{t}$ and $\underline{a} = \ddot{s}\underline{t} + \frac{\dot{s}^2}{\rho}\underline{n}$ respectively.

(b) A smooth wire in the form of an arch of the cycloid. is given by $s = 4a\sin\psi$, where $\frac{\pi}{2} \leq \psi \leq \frac{\pi}{2}$ and a is a positive constant. The wire is fixed in a vertical plane with its axis vertical and its vertex O at its lowest point. A bead P of mass m, moves under gravity on this wire. Given that the bead is projected from the vertex O with speed $2\sqrt{ag}$, show that when P reaches the point at which the tangent is inclined at an angle θ to the horizontal:

(i) its speed is $2\sqrt{ag}\cos\theta$.

(ii) the normal contact force exerted by the wire on the bead is $2mg\cos\theta$.

4. (a) With the usual notation show that the equation of the central orbit of a particle moving in a plane is given by

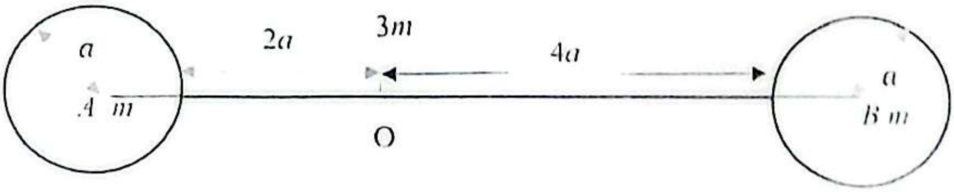
$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2} \quad \text{and} \quad \dot{\theta} = hu^2.$$

(b) A particle moves under the central force μ/r^3 per unit mass. It is projected from an apse at a distance a from the origin with velocity $\frac{2}{a}\sqrt{\frac{\mu}{3}}$. Show that the path is given by $r\cos(\theta/2) = a$.

5. (a) Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} + \underline{u}\frac{dm}{dt}$ for the motion of a particle of varying mass, moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, the matter being emitted at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

(b) A rocket has total mass M. It propels itself by burning fuel and ejecting the burnt matter at a uniform rate with constant speed u relative to the rocket. The total mass of the fuel in the rocket is initially $\frac{M}{2}$ and the fuel is all burnt up after a time T.
The rocket is launched from rest vertically upwards from the surface of the Earth. It may be assumed that the acceleration due to gravity remains constant throughout the flight of the rocket, and that air resistance is negligible. At time t the speed of the rocket is v.
(i) Show that, while the fuel is being burnt ($t<T$), $(2T - t)\frac{dv}{dt} = u - g(2T - t)$.
(ii) **Hence**, find the speed of the rocket at the instant when all the fuel has been burnt.

6.



A model of a timing device in a clock consists of two uniform circular discs, each of mass m and radius a, with a uniform rod attached as above. The centres of the discs are fixed to the ends A and B of the rod, which has mass $3m$ and length $8a$. The discs and the rod are coplanar, as shown in the figure. The body is free to rotate in a vertical plane about a smooth fixed horizontal axis. The axis is perpendicular to the plane of the discs and passes through the point O of the rod, where $OA = 3a$.

(a) Show that the moment of inertia of the body about the axis is $54ma^2$.
(b) Find the period of small oscillations of the system about its position of stable equilibrium.