The Open University of Sri Lanka

B.Sc. in Information Technology Degree Programme

ADE3430 – Foundational Mathematics

Final Examination - 2024/2025

Duration: - Three Hours

Date: 11. 06. 2025 Time: 9.30 a.m. - 12.30 p.m.

General Instructions

- This paper consists of **TWO** sections, Section A and Section B. Section A is **compulsory**, it consists of **TEN** questions. Answer **all** of them.
- Section B consists of SIX Essay Questions and answer only **FOUR** of them.

SECTION A

Answer all questions (i) – (x).

(i) Find the quotient and remainder when the polynomial $x^3 - x^2 - 5x - 3$ is divided by $x + 4$ using the Division Algorithm.

(ii) Two pens and five pencils cost $Rs.\ 220.00$. If a pen cost 5 rupees more than a pencil, find the cost of each.

(iii) Solve the inequality $\frac{x+1}{3x-6} > 0$.

(iv) Simplify $\frac{1}{4}[\ln 24 + 3\ln 3 + \ln 18 - \ln 9]$.

(v) Using the identity $cos^2\theta + sin^2\theta = 1$, prove that $1 + tan^2\theta = sec^2\theta$.

(vi) Evaluate $\tan 75°$ using the addition formula.

(vii) Nimal walks two complete rounds of a circular path which has a diameter of $100m$. How far Nimal has walked? ($\pi = 3.14$)

(viii) Differentiate the following functions w.r.t. x.

a) $y = (2x^2 - 5x + 1)(x^2 - 3x + 4)$

b) $y = \frac{5x-3}{2x+4}$

c) $y = \sin(x^2 + 3x)$

d) $y = e^{3x+5}$

(ix) Evaluate $\int (2x^2 + 5x + 3)\, dx$.

(x) Construct the frequency distribution table and draw the increasing cumulative frequency graph for the following data.

1, 1, 2, 5, 3, 2, 5, 3, 2, 0, 1, 2, 1, 2, 1, 0, 3, 4, 0, 0, 0, 1, 1, 1, 4

SECTION B

Answer FOUR questions ONLY.

1. i) Consider the quadratic function $y = x^2 - 10x + 16$.

   a) Find the roots of the function.

   b) Express the function in completed square form and sketch the graph of y as a function of x.

   ii) Express $\frac{5x+4}{(x+5)(x-2)}$ in partial fractions.

   iii) A captain and a vice captain are to be chosen from a group of 10 cricket players. Of the remaining 8 players, 3 will be selected to be the spin bowler, fast bowler, and wicket keeper. How many ways are there to allocate the 5 positions?

   iv) Consider the matrix $A = \begin{pmatrix} 2 & 1 \\ 3 & -4 \end{pmatrix}$. Find A^T, $\det(A)$, *and* A^{-1}.

2. i) Prove that $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$.

ii) Show that the points $(3,4), (3,6)$ and $(3+\sqrt{3}, 5)$ are the vertices of an equilateral triangle where each side is of length 2.

iii) Find the equation of the straight line that passes through the points $(2, 4)$ and $(-6, 8)$.

iv) a) Determine the equation of the circle with the center $(2,1)$ and the radius 5 units.

b) Find the radius and the co-ordinates of the center of circle,

$$x^2 + y^2 + 6x - 12y + 9 = 0.$$

3. i) Evaluate the following limit.

$$\lim_{x \to \infty} \frac{5x - 2}{6x + 9}$$

ii) a) Let $y(x) = 4x^4 + 7x^2 + 5$. Find the value of third derivative at $x = 1$.

b) Find $\frac{dy}{dx}$ given the parametric equations $x = \cos\theta$, $y = \sin\theta$.

iii) a) Find the equation of the tangent drawn to the curve,

$$y = 4x^2 - 5x + 7 \ \ at\ x = 3.$$

b) Find the relative maximum and minimum of the function

$$f(x) = 2x^3 + 3x^2 - 12x + 5.$$

iv) The perimeter of a rectangular land is 48m. When its area is maximum, find the length and the width of the land.

4. i) Find the following indefinite integrals.

a) $\int (x + x^4 - 1)\, dx$

b) $\int (1 + 2\sin x + 4e^x - \frac{1}{x})\, dx$

c) $\int 2x(x^2 + 3)^4\, dx$

d) $\int \frac{3x^2+2x+3}{x^3+x^2+3x}\, dx$

ii) Find the definite integral,

$$\int_0^1 6x(3x^2 - 2)^3\, dx.$$

iii) Given that $\int_6^{-10} f(x)dx = 23$ and $\int_{-10}^{6} g(x)dx = 9$, determine the value of $\int_{-10}^{6}[2f(x) - 10g(x)]dx$.

5. i) The number of books checked out from the library by 22 students are as follows:

1, 2, 3, 3, 4, 4, 5, 5, 7, 7, 7, 7, 8, 8, 8, 9, 10, 10, 11, 11, 12, 12

Find the mean of the books checked out from the library.

ii) Amal and his friends bought 8 burgers from 8 different shops. The prices of burgers differ from shop to shop as follows.

Rs. 790, 600, 290, 840, 960, 200, 750, 690.

Calculate the median cost of burgers.

iii) Find the quartile deviation of 8, 9, 12, 15, 14.

iv) If the mean is 9 and the variance is 92.5 for a data set, calculate the coefficient of variance.

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6. i) If A and B are any two sets show the following in Venn diagrams.

   a) $(A \cap B)'$

   b) $A' \cap B$

   ii) A box contains 16 marbles numbered 1, 2, 3,...,16. One marble is randomly selected. Let A be the event 'the marble selected is a prime number greater than 3' and let B be the event 'the marble selected is an odd number'. Evaluate the following.

   a) $P(A)$

   b) $P(B)$

   c) $P(A \cap B)$

   d) $P(A \cup B)$

   iii) A fair 6-sided die is rolled 48 times. How many times is an even number expected to show on the top?

   iv) Kamal is preparing for his exams. He has three upcoming exams, ICT, Mathematics, and English. The probabilities of passing ICT, Mathematics, English are 0.7, 0.5,0.8 respectively.

   a) Represent this information on tree diagrams.

   b) Assuming the events are independent, calculate the probability that he passes all three exams.

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