THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./ B.Ed. Degree Programme
APPLIED MATHEMATICS - LEVEL 03
ADU3302 - Differential Equations
NO BOOK TEST - 2024/2025
DURATION: ONE HOUR

Date: 16.03.2025 **Time: 4.00 p.m. to 5.00 p.m.**

ANSWER ALL QUESTIONS

1. Consider the differential equation

$$\frac{dy}{dx} + \frac{1}{2}y = \frac{1}{2}y^3.$$

(i) Use a suitable substitution to convert the above differential equation into the form

$$\frac{dz}{dx} + P(x)z = Q(x).$$

(ii) Use the integral factor method to solve the differential equation found in (i) above and hence find y in terms of x.

(iii) If $y(0) = \frac{1}{2}$, then find the solution of the originally given differential equation.

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2. A cup of hot tea is left to cool in a room with a constant ambient temperature T_0. According to Newton's Law of Cooling, the rate at which the temperature of the tea decreases is proportional to the difference between its temperature and the room temperature.

(i) Assuming that the cooling of tea in the cup follows this law, write down the differential equation in terms of T, T_0, t, and a proportionality constant k.

(ii) Show that the temperature of the tea at time t is given by

$$T = T_0 + Ae^{-kt}$$

where A is a constant.

(iii) Given that the initial temperature of the tea is $80°C$ and it cools to $60°C$ after 10 minutes, while the room temperature remains at $20°C$, determine the value of the proportionality constant k in Newton's Law of Cooling.

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