The Open University of Sri Lanka

B.Sc/B.Ed Degree programme

Pure Mathematics – Level 04

PEU4301- Real Analysis II

No Book Test (NBT)- 2024/2025

Date: 14.03.2025	**Time: 10.30 a.m. – 11.30 a.m.**

Answer All Questions.

1. Let $g(x) = \begin{cases} \sqrt{x}, & x \in (0,2] \\ 1, & x = 0 \\ -1, & x \in [-1,0). \end{cases}$

   (a) Prove that $\lim_{x\to 0^-} g(x) = -1$.

   (b) Is the $\lim_{x\to 0} g(x)$ exists? Justify your answer.

2. Below is the proof demonstrating that $\lim_{x\to\infty} \frac{x+2}{x^2+4} = 0$.

   Fill in the blanks to complete it. (You have the freedom to add additional lines if needed.)

   Let $\varepsilon > 0$.

   Notice that $\left|\frac{x+2}{x^2+4} - 0\right| = \frac{x+2}{x^2+4} \leq$ ..... (1), for .....(2)

   Let $b =$..... (3)

   Then $b \epsilon \mathbb{R}$.

   Now let $x > b$. Then ..... and ..... (4)

   ..... (5)

   ..... (6)

   Hence for each $x > b$, $\left|\frac{x+2}{x^2+4} - 0\right| < \varepsilon$.

   Thus $\lim_{x\to\infty} \frac{x+2}{x^2+4} = 0$.

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