The Open University of Sri Lanka

B.Sc/B.Ed Degree programme

Applied Mathematics – Level 03

ADU3302- Differential Equations

Final Examintaion - 2023/2024

Date: 31.03.2023	**Time: 09.30 a.m. – 11.30 a.m.**

General Instructions

- This paper consists of **TWO** sections, Section **A** and Section **B**. Section **A** is compulsory and it consists of **FIVE** Structured Essay Questions and carries 100 marks.
- Section B consists of **FIVE** essay-type questions and answer only **THREE** of them. Each question in Section B carries 100 marks.
- This paper consists of 03 pages.

SECTION A

1. **Answer all the questions in this section.**

(a) Determine whether the equation $\frac{dy}{dx} = -\frac{x}{y}$ is linear or non-linear.

(b) Verify the function $y(x) = e^{-3x}$ is a solution of the differential equation,

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 15y = 0.$$

(c) Find the value for *b* which the equation $ye^{2xy} + x + bxe^{2xy}y' = 0$ exact.

(d) Consider the second order differential equation $y'' + 2y' + 2y = 3e^{-t} + 2e^{-t}\cos t.$
Identify the *UC*-functions and generate the corresponding *UC*-sets.

(e) A ball of mass *m* is dropped with no initial velocity and encounters an air resistance that is proportional to the square of its velocity. Find an expression for the acceleration of the ball at a time *t*. Consider *v* as the velocity, and *g* as the gravitational force.

SECTION B

Answer **THREE** Questions **ONLY** from this section.

2. (a) Consider the differential equation $\frac{dy}{dt} = f\left(\frac{y}{t}\right)$.

   (i) Show that using a suitable substitution replaces the above equation by the equivalent equation $v + t\frac{dv}{dt} = f(v)$.

   (ii) `Find the general solution of the equation $\frac{dy}{dt} = 2\left(\frac{y}{t}\right) + \left(\frac{y}{t}\right)^2$.

   (b) Consider the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x-y+3}$.

   (i) Using the substitutions $x = X + h$ and $y = Y + k$ form an expression for $\frac{dY}{dX}$.

   (ii) Determine the h and k where the equation formed in part(i) can be formed in to a variable-separable equation.

3. A family of swordfish living off the Indian Ocean obeys the population growth $\frac{dp(t)}{dt} = 0.003p(t)$where t is measured in minutes.

   (i) An undesirable element has moved into their neighborhood, 0.002 swordfish per minute leave. Modify the given population growth equation by taking this factor into account.

   (ii) Assume that at time $t = 0$ there are one million swordfish. Find the population $p(t)$.

   (iii) Assume that, in addition to the incident mentioned in part (i), at time $t = 0$ a group of sharks establish residence in these waters and begin attacking the swordfish. The rate at which swordfish are killed by the sharks is $0.001p^2(t)$, where $p(t)$ is the population of swordfish at time t.
   Modify the given population growth equation in part(i) by taking this factor into account. Find the population $p(t)$.

4. (a) Find the general solution of the equation $(2y + e^t)dt + (2t + \cos y)dy = 0$.

   (b) Using a suitable integrating factor convert $\frac{dy}{dx} = 2xy - x$ into an exact differential equation.

   (c) Show that every separable equation of the form $M(x) + N(y)\frac{dy}{dx} = 0$ is exact.

5. (a) Find the general solution of the equation $y'' - 2y' - 2y = 0$.

(b) Find the particular solution of the equation $y'' - 2y' - 2y = -2t + 4t^2$ and then deduce the general solution.

(c) Using the D-operator method find the particular solution of the equation,

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 4y = e^{2x} + \sin 3x.$$

(Hint: Euler's formula for $\sin x$: $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$)

6. (a) Suppose that the temperature of a cup of tea obeys Newton's law of cooling. If the tea has a temperature of 200°F when freshly poured, and 1 min later has cooled to 190°F in a room at 70°F, determine when the tea reaches a temperature of 150°F.

(b) Determine whether $x = 0$ is a regular singular point of the differential equation

$$8x^2\frac{d^2y}{dx^2} + 10x\frac{dy}{dx} + (x - 1)y = 0.$$