The Open University of Sri Lanka
B.Sc. Degree Programme
Applied Mathematics - Level 04
Open Book Test-2017/2018
ADU4301/ADE4301 — Newtonian Mechanics I

DURATION: ONE HOUR

Date: 05. 01. 2019 **Time: 09.00 a.m. –10.00 a.m.**

ANSWER ALL QUESTIONS.

1. A smooth wire is in the shape of an arc of the cycloid $s = 2a\sin\psi$, $-\frac{\pi}{2} < \psi < \frac{\pi}{2}$. The wire is fixed in a vertical plane with its vertex, the origin O at its lowest point. A bead of mass m is threaded on the wire. At time t the bead is at the point P, the arc length $OP = s$, and the tangent at P makes angle ψ with the horizontal. The bead is released from rest at the point where $s = \frac{3a}{2.}$.

   (a) Show that $\ddot{s} = -\left(\frac{g}{2a}\right)s$.

   (b) Show that the speed of the bead at O is $\frac{3}{2}\sqrt{\left(\frac{ag}{2}\right)}$.

   (c) Find the magnitude of the normal reaction exerted on the bead by the wire as the bead passes through O.

2. A particle P of mass $2m$ kg is free to move on a smooth horizontal table. The particle is attached to one end of a light elastic string of natural length a and modulus of elasticity $4mg$ N. the other end of string is attached to a fixed point O on the table. The position of P is specified by polar coordinates (r, θ) referred to O as pole and the fixed line OA as initial line, where $OP = r$ meters. Initially P is held at a point on OA with the string just taut. It is projected horizontally at right angle to OA with a speed $3\sqrt{ag}$ ms^{-1}. Given that the string remains taut, Show that, t seconds after projection,

   (a) $\frac{d^2r}{dt^2} = \left(\frac{9a^3}{r^3} - \frac{2}{a}(r-a)\right)g$.

   (b) $\dot{r}^2 = \left(9a - \frac{9a^3}{r^2} - \frac{2}{a}(r-a)^2\right)g$

   (c) Hence show that, in the subsequent motion, the length of the string varies between a m and $3a$ m.