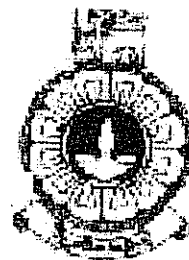


THE OPEN UNIVERSITY OF SRI LANKA  
M.A.IN TEACHER EDUCATION – PART II  
FINAL EXAMINATION 2004/2005  
PME 3497 – MATHEMATICS AT SECONDARY LEVEL



DURATION: THREE-HOURS

DATE: 26.05.2006

TIME: 10.00am – 1.00pm

ANSWER **THREE** QUESTIONS ONLY.

Non-programmable calculators are permitted.

1. The rules for a game are as follows. A coin is tossed. If a head appears the score of the game is 10. If the coin lands tail, the player is asked to roll a fair die with sides marked 1, 2,3,4,5 and 6 and the number turning up is taken as the score of the game.

Let  $A$  be the event that the score of the game is even. Let  $B$  be the event that the score of the game is greater than 2.

- i) Write down the sample space.
  - ii) Compute the probabilities of the events  $A$  and  $B$ .
  - iii) Are the events  $A$  and  $B$  independent? Give reasons for your answer.
  - iv) Compute the conditional probability of  $A$  given that the event  $B$  has occurred.
  - v) Let  $X$  be the random variable that denotes the score of the game. Write down the probability distribution of  $X$ .
  - vi) Compute the expected value of  $X$  and interpret it.
- 4) i) Use truth tables to prove that the statements  $p \Rightarrow q$  and  $(\sim q) \Rightarrow (\sim p)$  are equivalent. Here  $(\sim q)$  implies the negation of the statement  $q$ .
- ii) Using the results of part (i) or otherwise prove that if  $n$  is a natural number such that  $n^2$  is even, then  $n$  itself is even.
- iii) Prove by mathematical induction that, if  $n$  is a positive integer, the sum  $n.1 + (n-1)2 + (n-2)3 + \dots + 2(n-1) + 1.n$  is equal to  $\frac{1}{6}n(n+1)(n+2)$ .

- 3) A student is interested in finding out the whether the final examination marks ( $y$ ) are correlated with the continuous assessment marks ( $x$ ) for a particular course and if so to predict the final examination marks using the continuous assessment marks. The following summary statistics were obtained from the final examination marks and continuous assessment marks of 20 students.

$$\sum_{i=1}^{20} x_i = 949, \sum_{i=1}^{20} y_i = 1019, \sum_{i=1}^{20} x_i^2 = 53615, \sum_{i=1}^{20} y_i^2 = 62227, \sum_{i=1}^{20} x_i y_i = 57465.$$

- i) Compute the correlation coefficient between the continuous assessment marks and final examination marks and interpret it.
  - ii) Obtain the equation of the regression line using least squares.
  - iii) The continuous assessment marks of two students differ by only 1 mark. Give an estimate for the difference in their final examination marks?
  - iv) Use the regression line obtained in part (ii) to predict the final examination mark of a student who has obtained a continuous assessment mark of 61.
  - v) Two students have obtained the same continuous assessment mark. Is it possible for them to have different final examination marks? If not, give reasons. If yes, how do you explain that difference?
- 4) a) A radioisotope is decaying at a rate proportional to the amount of isotope  $x(t)$  present at time  $t$ .
- i) Write down a differential equation satisfied by  $x(t)$ .
  - ii) Assume that initially there was 1.0 mg of isotope. Find the time it takes to reduce the amount to 0.5 mg. Assume that the weight is only reduced by the decay.
- b) i) Show that  $2^n$  and  $n2^n$  are both solutions of the difference equation  $f(n+2) - 4f(n+1) + 4f(n) = 0$ ;
- ii) Find a difference equation of which the function  $f(n) = (A+n)n!$ , where  $A$  is an arbitrary constant, is a solution.
- 5) a) A deposit  $A$  is invested at a monthly compound interest rate of  $r\%$ . This means, at the end of each interest period, the interest earned also gets added to the initial deposit thus earning interest on the interest earned as well. Show that the total interest earned after  $t$  months is given by  $A(1+r)^t - A$ .
- b) A person invests Rs. 100000/- at 8% interest rate compounded monthly for a period of 3 years. At the end of the three years period what would be the interest earned?
- c) If the person invests Rs. 100000/- at 10% simple interest rate how long will the person have to wait to earn the same interest?