THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF INDUSTRIAL STUDIES

TTZ5244 QUANTITATIVE TECHNIQUES

FINAL EXAMINATION - 2012/2013

DURATION - THREE HOURS

Date: 22nd August 2013 **TIME: 0930 - 1230 Hours**

This question paper consists of three sections A, B and C. Answer all the questions in Section A and two (02) questions each from Sections B and C. [Total questions to be answered are 05].

Section A carries 20 marks and Sections B & C carry 40 marks each.

You should clearly show the steps involved in solving problems. No marks are awarded for mere answers without writing the necessary steps

SECTION A

This section carries 20 marks. Answer all questions in this section.

1. Solve the following equations **(02 marks)**

   (i) $5^{3x+1} + 12 = 637$ (ii) $3^{4x+1} - 8 = 235$

2. If $P = \dfrac{iA}{1-(1+i)^n}$

   Obtain an expression for 'n' in terms of P, A and i **(02 marks)**

3. What do you understand by $\left[\dfrac{dy}{dx}\right]$, if y is a function of x? **(02 Marks)**

4. Determine the second derivative of the following functions with respect to x **(02 marks)**

(i) $y = 12x^4 + 8x^3 + 5x + 20$ (ii) $y = e^{(5x+8)}$

5. Define the "gradient "and the "intercept" of a straight line graph. **(02marks)**

6. Determine the stationary points of the following function and find whether they are minima or maxima **(04 Marks)**

$$y = 12x^3 - 4x + 5$$

7. Give an example for Skew-symmetric matrix **(03 marks)**

8. Find the determinant of matrix A **(03 marks)**

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 2 & 6 \\ 2 & 4 & 3 \end{pmatrix}_{3x3}$$

SECTION B

Maximum possible mark for this section is 40. Answer any two questions from this section. Each question in this section carries 20 marks.

Question 9

A company incurs a fixed production cost of Rs. 1280/= and a variable cost of Rs 80/= per unit output. Its demand function is $P = 100 - Q/20$, where P is the unit price and Q is the number of units of demand.

(i) Write an equation for the total cost of production **(03 marks)**

(ii) Express revenue as a function of Q (Revenue 'R' is given by $R = PQ$) **(04 marks)**

(iii) Express the total profits as a function of Q (Profit = revenue – total cost) **(04 marks)**

(iv) Sketch the 'profit graph' as a function of Q. **(06 marks)**

(v) How many units should be produced in order to maximise the profit. **(03 marks)**

Question 10

a. Differentiate the following functions with respect to X

(i) $Y = (5x - 3)(6X^2 + 8X + 10)$

(ii) $Y = \log_e (5X^2 + 4X + 4)$ **(04 Marks)**

b. If the "marginal revenue function" is defined as $\frac{dR}{dQ}$, where R is the revenue and Q is the demand. If Q and R are related by,

$$R = 20\,Q - 0.001\,Q^2$$

what is the marginal revenue, when $Q = 2000$? **(08 Marks)**

c. Price elasticity of demand, E is defined as,

$$E = -\frac{P}{Q}\frac{dQ}{dP}$$ where Q is the demand and P is the price

If the demand function is $Q = 100 - P^2$, what is the price elasticity of demand (E) when P = 3?

(08 Marks)

Question 11

Solve the following systems of linear equations using matrix inversion.

$$2X + 2Y - 6Z = 4$$

$$-X + Y + 2Z = 6$$

$$-3X + 5Y + 3Z = -1$$

(20 Marks)

Question 12

(a) If,

$$A = \begin{pmatrix} 5 & 2 \\ 3 & 7 \end{pmatrix}_{2x2} \qquad B = \begin{pmatrix} 6 & 4 \\ 2 & 1 \end{pmatrix}_{2x2}$$

Determine A^2 and AB **(04 marks)**

(b) If,

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}_{3x3}$$

Find the inverse of the matrix A **(08 Marks)**

(c) If,

$$A=\begin{bmatrix} 1, & 0 \\ -1, & 3 \end{bmatrix}, \quad I=\begin{bmatrix} 1, & 0 \\ 0, & 1 \end{bmatrix},$$

Show that $A^3 - I = (A - I)(A^2 + A + I)$ **(08 Marks)**

SECTION C

Maximum possible mark for this section is 40. Answer any two questions from this section. Each question in this section carries 20 marks.

Question 13

A factory produces two types of soft toys: Model A and Model B. Both products are processed in three machines M_1, M_2, and M_3.The factory can make a profit of Rs. 80/= on model A and Rs. 120/= from model B.. The time required in hours to manufacture each model on each of the machines and the total time available in hours per month on each of the machines is given in the table below.

Machine	Manufacturing time (in hours)		Available time in hours
	Model A	Model B	
M_1	2	1	180
M_2	1	2	160
M_3	1	1	100

Formulate a linear programming model for this situation.

a. Name the variables in this problem. **(02 marks)**

b. What are the constraints of the problem? **(04 marks)**

c. Solve the **formatted programme graphically** to determine how the factory should schedule production in order to maximise the profit. **(14 marks)**

Question 14

Winter Garments Pvt Ltd manufactures two different garments A and B .The profit contribution of A is Rs 4/= and B is Rs 5/=.The resources required to manufacture one unit of each product are shown below.

Resources	Garment A	Garment B
Fabric required in metres	2	1
Machine time in hours	1	2

During the next month only 1400 machine hours and 1600 metres of fabric are available for the production. The company's management team would like to determine the number of units of each product to be manufactured in order to maximise total profit contribution.

(a) Formulate the problem as a linear programming model. **(06 Marks)**

(b) Determine using **Simplex method,** how many of each garments to be manufactured in order to maximise the profit **(14 Marks)**

Question 15

A company manufacturing air conditioner components has firm orders for the next six (6) months in the future. The company can schedule its production over the next 6 months to meet the order on either regular or overtime basis. Orders and the associated production costs for the next 6 months are as follows:

Month	September	October	November	December	January	February
Orders	550	600	620	580	700	710
Cost /unit (Rs) (Regular production)	50	55	58	52	60	62
Cost /unit (Rs) (Overtime production)	60	63	65	58	67	68

With 300 components in stock in the beginning of September, the company wishes to have at least 550 of them in stock at the end of February. The inventory carrying cost for these components is Rs 5/= per unit per month. If the regular and overtime production in each month is not to exceed 500 components and 400 components respectively,

Formulate linear programme for the problem.

(i) What are the variables in this problem? **(06 marks)**

(ii) Write the objective of this problem. **(07 marks)**

(iii) What are the constraints of the problem? **(07 marks)**