THE OPEN UNIVERSITY OF SRILANKA
B.Sc / B.Ed DEGREE PROGRAMME – LEVEL 05
OPEN BOOK TEST (OBT)
APPLIED MATHEMATICS
ACADEMIC YEAR 2004/2005
AMU 3182/ AME 5182 – MATHEMATICAL METHODS I
DURATION – **ONE AND HALF HOURS**

Date: 17 – 09 – 2005 Time: 10.30am to 12.00noon

Answer all questions.

1. (a) Solve the following differential equations.

 (i) $y'' + 9y = 6\cos 3x$

 subject to $y = \pi/4$ when $x = 0$ and, $y = \pi/3$ when $x = \pi/6$.

 (ii) $x^2 u''(x) - xu'(x) - 3u(x) = 0, \qquad x > 0$

 u(1) = 1 and $\lim_{x \to \infty} u(x)$ is bounded.

 (iii) $\dfrac{d^2y}{dx^2} + y + 2x^2 = 3$, given that $y = 7$ and $\dfrac{dy}{dx} = 0$ when $x = 0$.

 (b) Show that general solution of the equation

$$\frac{d^2x}{dt^2} + n^2 x = 0$$

 can be written in the form

 $C\,Sin(nt + \alpha)$,where C, α are arbitrary constants.

 If $n = 6$ and $t = 0, x = \dfrac{9\sqrt{3}}{2}, \dfrac{dx}{dt} = 27$ find C, α.

2. (a) Transform the equation

$$(1 + x^2)^3 \frac{d^2y}{dx^2} + 2x(1 + x^2)^2 \frac{dy}{dx} + (1 + x^2)y = 3x$$

 by the substitution $x = \tan\theta$.

 Hence or otherwise, determine the solution of this equation for which both y and $\dfrac{dy}{dx}$ vanishes when $x = 0$.

(b) (i) Write down, in a form suitable for generating solutions, the recurrence relations corresponding to an application of Euler's method in respect of

$$\frac{dx_1}{dt} = 3tx_2 + 4$$

$$\frac{dx_2}{dt} = tx_1 - x_2 - e^t \qquad \text{where } x_1 = 5, x_2 = 2 \text{ at } t = 0.$$

(ii) Use the recurrence relations you obtain in (i) with step length 0.1 to calculate $x_1(0.2)$ and $x_2(0.2)$.

3. Find the solution of each of the systems of equations given below in the usual notation.

(i)
$$\dot{x} - 2x + 2\dot{y} = 2 - 4e^{2t}$$
$$2\dot{x} - 3x + 3\dot{y} - y = 0$$

(ii)
$$\ddot{y}_1 = 3y_1 + 2(y_2 - y_1)$$
$$\ddot{y}_2 = -2(y_2 - y_1)$$