The Open University of Sri Lanka
B.Sc. B.Ed. Degree Programme – Level 05
Open Book Test (OBT) – 2004/2005
Applied Mathematics
AMU 3184/AME 5184 – Dynamics

Duration :- One and Half Hours

Date :- 03-09-2005 **Time :- 4.00 p.m. – 5.30 p.m.**

Answer All Questions.

01. A bead of weight 3m is free to slide on smooth horizontal straight wire. A particle of mass m is attached to the bead by a light inelastic string of length l. When the particle is released, the string is tight and the particle is touching the wire. If the angle between the wire and the string is θ and the displacement of the bead along the wire is x at time t. Show that $x = \frac{l}{4}(1 - \cos\theta)$.

Further show that the angular velocity is given by $(3 + \cos^2\theta)\omega^2 l = 8g\sin\theta$.

02. A smooth wire in the form of $S = 4a\sin\psi$, $\left(-\frac{\pi}{2} < \psi < \frac{\pi}{2}\right)$ is fixed in a vertical plane, the vertex O being the lowest point of the wire. A bead of mass m, which can slide freely on the wire is released from rest at the point where $\psi = \frac{\pi}{6}$.

Find the period of oscillation of the bead, and show that the reactions of the wire at a point where the tangent makes an angle ψ with the horizontal is $mg\sec\psi(8\cos^2\psi - 3)/4$.

03. In a spherical polar coordinate system, let $\underline{a}$, $\underline{b}$, $\underline{c}$ be unit vectors defined as follows:

$\underline{a}$ = unit vector in the direction of increasing r when θ, ϕ are fixed.

$\underline{b}$ = unit vector in the direction of increasing θ when r, ϕ are fixed.

$\underline{c}$ = unit vector in the direction of increasing ϕ when r, θ are fixed.

Use the fact that $\underline{a}$, $\underline{b}$, $\underline{c}$ are mutually orthogonal unit vectors to show that there is a skew-symmetric matrix $[p_{ij}]_{3\times3} = \underline{P}$ such that

$$\begin{pmatrix}\dot{\underline{a}}\\ \dot{\underline{b}}\\ \dot{\underline{c}}\end{pmatrix} = \underline{P}\begin{pmatrix}\underline{a}\\ \underline{b}\\ \underline{c}\end{pmatrix}$$

Using the values of the angles made by $\underline{c}$ with each of the axes Ox, Oy, Oz, show that

$$p_{12} = \dot{\theta}, \quad p_{13} = \dot{\phi}\sin\theta, \quad p_{23} = \dot{\phi}\cos\theta.$$

Hence, show that, the velocity vector is given by $\dot{\underline{r}} = \dot{r}\underline{a} + r\dot{\theta}\underline{b} + r\dot{\phi}\sin\theta\underline{c}$.

Deduce the acceleration vector is given by

$$\ddot{\underline{r}} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\underline{a} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta\right)\underline{b}$$

$$+\left(2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + r\ddot{\phi}\sin\theta\right)\underline{c}.$$