DATE:22/02/2006   TIME:4.00pm-5.30pm

ANSWER ALL QUESTIONS

1. (a) Let F be the set of all continuous functions $f : \mathbb{R} \to \mathbb{R}$. Then show that F forms a ring under addition and multiplication defined by : for any $f, g \in F$

$$(f + g)(x) = f(x) + g(x) \qquad , \forall x \in \mathbb{R}$$

$$(fg)(x) = f(x)g(x) \qquad , \forall x \in \mathbb{R}$$

(b) Show that if an element $a \in \mathbb{Z}_n$ is a unit then a and n are relatively prime.

(c) If in a ring R with unity $(xy)^2 = x^2 y^2$ for all $x, y \in R$ then show that R is commutative.

2. (a) Show that a non zero idempotent cannot be nilpotent.

(b) Find all the idempotent and nilpotent elements of the ring $\mathbb{Z}_4$.

3. (a) Let R be the ring of 3×3 matrices over real numbers. Show that

$$S = \left\{ \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix} \mid x \in \mathbb{R} \right\} \text{ is a subring of } R \text{ and has unity different from unity of } R.$$

(b) Let R be a ring of 2×2 matrices over integers. Is $A = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a, b \in \mathbb{Z} \right\}$ a both sided ideal? Justify your answer.

(c) "A subring may not be an ideal". Is this statement true or false? Justify you answer.