THE OPEN UNIVERSITY OF SRI LANKA B.Sc./B.Ed. Degree Programme, Continuing education Programme LEVEL 5- PURE MATHEMATICS PME 5295/PMU 3295 RING THEORY OPEN BOOK TEST-2004/2005



DURATION: ONE AND HALF HOURS

DATE:22/02/2006

TIME:4.00pm-5.30pm

ANSWER ALL QUESTIONS

1. (a) Let F be the set of all continuous functions $f: \mathbb{R} \to \mathbb{R}$. Then show that F forms a ring under addition and multiplication defined by: for any $f, g \in F$

$$(f+g)(x) = f(x) + g(x)$$
, $\forall x \in \mathbb{R}$
 $(fg)(x) = f(x)g(x)$, $\forall x \in \mathbb{R}$

- (b) Show that if an element $a \in \mathbb{Z}_n$ is a unit then a and n are relatively prime.
- (c) If in a ring R with unity $(xy)^2 = x^2y^2$ for all $x, y \in R$ then show that R is commutative.
- 2. (a) Show that a non zero idempotent cannot be nilpotent.
 - (b) Find all the idempotent and nilpotent elements of the ring \mathbb{Z}_4 .
- 3. (a) Let R be the ring of 3×3 matrices over real numbers. Show that $\begin{pmatrix} x & x & x \end{pmatrix}$

$$S = \left\{ \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix} \middle| x \in \mathbb{R} \right\} \text{ is a subring of } R \text{ and has unity different from unity of } R.$$

- (b) Let R be a ring of 2×2 matrices over integers. Is $A = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a, b \in \mathbb{Z} \right\}$ a both sided ideal? Justify your answer.
- (c) "A subring may not be an ideal". Is this statement true or false? Justify you answer.