THE OPEN UNIVERSITY OF SRI LANKA

B.Sc/B.Ed Degree Programme, Continuing Education Programme

APPLIED MATHEMATICS - LEVEL 05

AMU3189/AME 5189 - STATISTICS II

OPEN BOOK TEST 2004/2005

DURATION: ONE AND HALF-HOURS

| DATE: 30 – 01 – 2006 | TIME: 4.00 pm – 5.30 pm |
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**Non-programmable calculators are permitted.**

ANSWER **ALL** QUESTIONS.

1. Let $X_1, X_2, \cdots, X_n$ be a random sample from a uniform distribution so that the probability density function is given by

$$f(x,\theta) = \begin{cases} \dfrac{1}{1+\theta} & -1 \le x \le \theta \\ 0 & \text{otherwise} \end{cases}$$

a) Find the mean and variance of $X$.

b) Find an estimator for $\theta$ using method of moments.

c) Find the mean squared error of the estimator you found in part (b) above.

d) Let {-0.2, 1.4, 0.8, -0.4, 1.3, 0.2, -0.3, 1.1, -0.4, 1.5} be a random sample from the above distribution.

(i) Compute the method of moments estimator for $\theta$ based on this sample.

(ii) Give an estimate for the mean squared error of the moment estimator computed in part (i) above.

2. Let $X_1, X_2, \cdots, X_n$ be a random sample from the Bernoulli distribution with parameter $\theta$ so that the probability density function is given by

$$f(x;\theta) = \begin{cases} \theta^x (1-\theta)^{1-x} & x = 0,1 \quad 0 \le \theta \le 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Prove that the density function $f(x,\theta)$ belongs to the exponential family. Hence or otherwise show that $T = \sum_{i=1}^{n} X_i$ is a sufficient statistic for $\theta$.

b) Using the sufficient statistic $T$ find an unbiased estimator for $\theta$.

c) Is the estimator found in part (b) above consistent for $\theta$? Give reasons for you answer.

d) Using the Cramer-Rao lower bound for the variance of an unbiased estimator, verify that the estimator found in part (b) above is the uniformly minimum variance unbiased estimator (UMVUE) for $\theta$.

3. The random variable $X$ denotes the number of trains arriving at a certain railway station during 8.00am to 9.00am. Suppose $X$ has a Poisson distribution with parameter $\lambda$. Let $X_1, X_2, \cdots, X_n$ denote the number of trains arrived during 8.00am to 9.00am on $n$ randomly chosen days.

a) Find the mean and variance of $X$.

b) Find the maximum likelihood estimator of the expected number of trains arriving at this railway station during 8.00am to 9.00am.

c) Find an unbiased estimator for the probability that no train would arrive at the railway station during 8.00am to 9.00am.

d) The number of trains arrived during 8.00am to 9.00am on 8 randomly chosen days are found to be {2,3,0,4,0,2,3,2}. Compute the values of the estimators derived in parts (b) and (c) based on the given sample.