**The Open University of Sri Lanka**
**B.Sc. Degree Programme – Level 05**
**Closed Book Test (CBT) - 2009/2010**
**Pure Mathematics / Computer Science**
**PMU 3294/PME 5294/CSU 3276 – Discrete Mathematics**

**Duration: - One & Half Hours**

**Date: - 29-10-2009.** **Time: - 4.00 p.m. – 5.30 p.m.**

**Answer All Questions.**

01. (a) Define a conditional probability.

(b) Let $(S, P)$ be a probability space and suppose that $A_1, A_2, A_3$ be three events in $(S, P)$. Prove that $P(A_1 \cap A_2 \cap A_3) = P(A_1)\, P\left(A_2 / A_1\right) P\left(A_3 / A_1 \cap A_2\right)$.

(c) (i) What is the conditional probability that a family of three children has more than one boy, given that they have at least one boy.

(ii) If the condition for part (i) is "the first child is a girl", then what is the conditional probability that they have more than one boy.

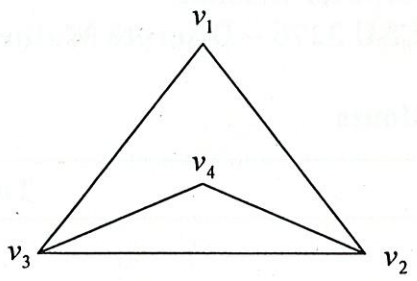
(iii) Determine whether the events "more than one boy out of three children" and "the first child is a girl" are independent.

02. (a) Define the degree of a vertex of a graph.

(b) Prove that $\sum_{i=1}^{n} \delta(v_i) = 2 \times (\text{Number of edges in the graph})$.

(Hint: $\delta(v_i) = \sum_{j=1}^{n} a_{ij}$ where $n$ is the number of vertices of the graph and $a_{ij}$ is the $(i, j)^{\text{th}}$ entry of the adjacency matrix of the graph)

(c) Give the set theoretic definition of the following graph $G$:




(i) By using the above graph $G$, verify the theorem in part (b),

(ii) Write down the adjacency matrix of the graph $G$,

(iii) Determine the number of paths of length three joining $v_1$ and $v_4$.

What are those paths?,

(iv) Using the matrix obtained in part (ii), show that $G$ is connected,

(v) Is $G$ a tree? Justify your answer.

03. (a) Define a second order homogeneous difference equation with constant coefficients.

(b) If $g(n)$ and $h(n)$ are solutions of a second order homogeneous linear difference equation with constant coefficients, then for any constants $\alpha$ and $\beta$, show that $\alpha g(n)+\beta h(n)$ is also a solution.

(c) Solve the following homogeneous difference equations:

(i) $2f(n+3)+3f(n+2)-f(n)=0$,

(ii) $f(n+2)-6f(n+1)+13f(n)=0$.