The Open University of Sri Lanka
B.Sc. Degree Programme – Level 05
Open Book Test (OBT) - 2009/2010
Pure Mathematics / Computer Science
PMU 3294/PME 5294/CSU 3276 – Discrete Mathematics

Duration: - One & Half Hours

Date: - 02-10-2009. **Time: - 4.00 p.m. – 5.30 p.m.**

Answer All Questions.

01. (a) Write the converse, inverse and contra-positive of the statement
"*If two triangles are similar then they are congruent*".

(b) Let p and q be two statements. Then by means of truth table, show that $(p \Leftrightarrow q)$ is logically equivalent to $(\neg p \vee q) \wedge (p \vee \neg q)$.

(c) Prove or disprove each of the following statements:

(i) $\forall x \in \mathbb{R},\ x^2 > 0.$

(ii) $\left(1^3+2^3+3^3+\ldots+n^3\right)=(1+2+3+\ldots+n)^2.$

(iii) $\forall x \in \mathbb{Q},\ x^2 \in \mathbb{Q}.$

02. (a) Which of the following relations are partial order:

(i) $R=\left\{(x, y)\,\middle|\,(x \in \mathbb{R}) \wedge (y \in \mathbb{R}) \wedge \left(y = x^2\right)\right\}.$

(ii) $R=\left\{(x, y)\,\middle|\,(x \in A) \wedge (y \in A) \wedge (\text{"}x \text{ divides } y\text{"})\right\},$
where $A = \{1, 2, 3, 4, 6\}.$

(iii) $R=\left\{(x, y)\,\middle|\,(x \in \mathbb{R}) \wedge (y \in \mathbb{R}) \wedge (y \leq x)\right\}.$

Justify your answer.

(b) Let f be a homomorphism from a group G into a group G'. Suppose that x^{-1} is the inverse of x in G. Prove that $f(x^{-1})$ is the inverse of $f(x)$ in G'.

(c) Determine whether the operation "*subtraction*" in $\mathbb{Z}$, the set of integers, is associative:

03. (a) The internal telephone numbers in the phone system on a campus consist of **five** digits, with the first digit not equal to zero. How many different numbers can be assigned in this system?

(b) Let the number of r - permutations of n objects is denoted by ${}^nP_r$ and the number of r - objects subsets of n - objects set is denoted by ${}^nC_r$. Then prove that $r!\,{}^nC_r = {}^nP_r$.

(c) There are 20 Mathematics students and 15 Computer Science students in the Discrete Mathematics class. Find the number of ways of selecting **12 students** from the class, if

(i) there are no restrictions,

(ii) all must be mathematics major,

(iii) all must belong to the same discipline,

(iv) the two disciplines must have the same number of representatives.