The Open University of Sri Lanka
B.Sc. Degree Programme – Level 05
Open Book Test (OBT) - 2009/2010
Pure Mathematics / Computer Science
PMU 3294/PME 5294/CSU 3276 – Discrete Mathematics

Model Answer

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01. (a) The given statement is : "*If two triangles are similar then they are congruent*".

Converse: "*If two triangles are congruent then they are similar*",

Inverse: "*If two triangles are not similar then they are not congruent*",

Contra - positive: "*If two triangles are not congruent then they are not similar*".

(b) Let p and q be two statements. Then consider the following truth table:

p	q	$p \Leftrightarrow q$	$\neg p$	$\neg q$	$(\neg p \vee q)$	$(p \vee \neg q)$	$(\neg p \vee q) \wedge (p \vee \neg q)$
T	T	T	F	F	T	T	T
T	F	F	F	T	F	T	F
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T

Truth values in the column under the heading $(p \Leftrightarrow q)$ are the same as truth values in the column under the heading $(\neg p \vee q) \wedge (p \vee \neg q)$.

Hence $(p \Leftrightarrow q)$ is logically equivalent to $(\neg p \vee q) \wedge (p \vee \neg q)$.

(c) (i) "$\forall x \in \mathbb{R},\ x^2 > 0$" is a false statement.

Since $x = 0\ (\in \mathbb{R})$, but $x^2 = 0$ is not greater than 0.

(ii) "$\left(1^3 + 2^3 + 3^3 + \dots + n^3\right) = (1 + 2 + 3 + \dots + n)^2$" is a true statement.

We will prove this by method of induction

We know that $\left(1^3+2^3+3^3+\ldots+n^3\right)=(1+2+3+\ldots+n)^2=\left[\frac{n(n+1)}{2}\right]^2$.

When $n=1$:

$1^3=1^2$ is true, therefore the result is true for $n=1$

Assume that the result is true for $n=k$:

i.e. $\left(1^3+2^3+3^3+\ldots+k^3\right)=(1+2+3+\ldots+k)^2=\left[\frac{k(k+1)}{2}\right]^2$

Now consider the result when $n=k+1$:

$$\begin{aligned}\left(1^3+2^3+3^3+\ldots+k^3+(k+1)^3\right)&=\left[\frac{k(k+1)}{2}\right]^2+(k+1)^3\\&=\frac{k^4+2k^3+k^2}{4}+k^3+3k^2+3k+1\\&=\frac{k^4+6k^3+13k^2+12k+4}{4}\\&=\left[\frac{(k+1)(k+2)}{2}\right]^2\\&=[1+2+3+\ldots+k+(k+1)]^2\end{aligned}$$

So the result is true for $n=k+1$ also

Therefore by principle of mathematical induction the result is true $\forall n\in\mathbb{N}$.

(iii) "$\forall x\in\mathbb{Q},\ x^2\in\mathbb{Q}$" is a true statement.

If $x\in\mathbb{Q}$, then $x=\frac{p}{q}$ where $p,q\in\mathbb{Z}$ with $q\neq 0$

So $x^2=\frac{p^2}{q^2}\in\mathbb{Q}$ $\quad(\because p,\ q\in\mathbb{Z}$ therefore $p^2,\ q^2\in\mathbb{Z}\ q^2\neq 0)$

In Addition, suppose if the question: 'is the statement

"$\forall x\in\mathbb{R}\setminus\mathbb{Q},\ x^2\in\mathbb{R}\setminus\mathbb{Q}$" true or false ?'.

Then it is a false statement.

Since $\sqrt{2}\in\mathbb{R}\setminus\mathbb{Q}$ but $\left(\sqrt{2}\right)^2=2\notin\mathbb{R}\setminus\mathbb{Q}$

02. (a) (i) $R=\{(x, y) \mid (x\in\mathbb{R})\wedge(y\in\mathbb{R})\wedge(y=x^2)\}$ is not a partial order

Since R is not reflexive $(\because$ there is no $_2R_2$ i.e. $2\neq 2^2)$

(ii) $R=\{(x, y) \mid (x\in A)\wedge(y\in A)\wedge(\text{"}x \text{ divides } y\text{"})\}$,

where $A=\{1, 2, 3, 4, 6\}$.Here

$$R=\begin{Bmatrix}(1, 1),(1, 2),(1, 3),(1, 4),(1, 6),(2, 2), \\ (2, 4),(2, 6),(3, 3),(3, 6),(4, 4),(6, 6)\end{Bmatrix}$$

and R is reflexive, anti- symmetric and transitive, therefore R is a partial order.

(iii) $R=\{(x, y) \mid (x\in\mathbb{R})\wedge(y\in\mathbb{R})\wedge(y\leq x)\}$ is a partial order

$_xR_x\ \forall x\in\mathbb{R}$ i.e. $x\leq x\,\forall x\in\mathbb{R}$, so R is reflexive

If $_xR_y$ and $_yR_x$, then $x\leq y$ and $y\leq x$ this gives $x=y$

So R is anti-symmetric

If $_xR_y$ and $_yR_z$, then $x\leq y$ and $y\leq z$ this gives $x\leq z$

So R is transitive and therefore R is a partial order.

(b) Let f be a homomorphism from a group $(G, *)$ into a group $(G', *')$.

Let e, e' be the identity elements of G and G' respectively.

Then $f(e)=e'$,

Let $x\in G$ and suppose that x^{-1} is the inverse of x in G.

Then $f(x*x^{-1})=f(x)*'f(x^{-1})$. Also

$$x*x^{-1}=e$$
$$f(x*x^{-1})=f(e)$$
$$f(x)*'f(x^{-1})=e' \quad (\because f \text{ is a homomorphism})$$

This gives $f(x^{-1})$ is the inverse of $f(x)$

i.e. $[f(x)]^{-1}=f(x^{-1})$.

(c) The operation "*subtraction*" in $\mathbb{Z}$, the set of integers, is not associative:

Since

$$\mathbb{Z} = \{....-2, -1, 0, 1, 2, ...\}$$

$$(12-6)-2 = 6-2 = 4 \quad \text{but} \quad 12-(6-2) = 12-4 = 8$$

03. (a) Digits are 0,1,2,3,4,5,6,7,8,9 and number of digits is 10

9	10	10	10	10

So the number of telephone numbers can be assigned is 9×10^4.

(b) Let the number of r - permutations of n objects is denoted by ${}^nP_r$ and the number of r - objects subsets of n - objects set is denoted by ${}^nC_r$.

Number of r - subsets of an n - objects set $= {}^nC_r = \dfrac{n!}{r!(n-r)!}$

Number of r - permutations of r objects $= r!$

Then by the product principle, the number of r - permutations of n objects is

$$r!\,{}^nC_r = r!\frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!} = {}^nP_r.$$

(c) There are 20 Mathematics students and 15 Computer Science students in the Discrete Mathematics class and the number of ways of selecting **12 students** from the class, if

(i) there are no restrictions $= {}^{35}C_{12}$,

(ii) all must be mathematics major $= {}^{20}C_{12}\,{}^{15}C_0$,

(iii) all must belong to the same discipline $= {}^{20}C_0\,{}^{15}C_{12} + {}^{20}C_{12}\,{}^{15}C_0$,

(iv) the two disciplines must have the same number of representatives $= {}^{20}C_6\,{}^{15}C_6$.