

The Open University of Sri Lanka
 B.Sc/B.Ed. Degree Programme
 No Book Test (NBT) - 2017/2018
 Pure Mathematics - Level 05
 PEU5303-Number Theory
 Duration: - One hour



Date: 09.02.2019

Time: 10:30 a.m. – 11:30 a.m.

Answer All Questions.

1. (a) Prove that following set is a complete set of residue modulo 5.
 $\{ 2^{40}, 9^{15}, 10^{10}, 7^7, 8^5 \}$.
 - (b) Write down all the integers r such that $1 \leq r < 12$ and $\gcd(r, 12) = 1$. Hence find whether each of the following sets is reduced residue system modulo 12 or not. Justify your answer.
 - (i) $\{13, 23, 53\}$.
 - (ii) $\{13, 23, 53, 33\}$.
 - (iii) $\{13, 23, 53, 103\}$.
 - (iv) $\{13, 23, 53, 33, 103\}$.
 - (c) Solve the following set of simultaneous congruences.

$$\begin{aligned} x &\equiv 5 \pmod{6} , \\ x &\equiv 4 \pmod{11} , \\ x &\equiv 3 \pmod{17} . \end{aligned}$$
2. (a) Let $M = 10^4a + 10^3b + 10^2c + 10d + e$, where a, b, c, d and e are integers. Prove the followings.
 - (i) $4|M$ if and only if $4|(10d + e)$,
 - (ii) $9|M$ if and only if $9|(a + b + c + d + e)$,
 - (iii) $11|M$ if and only if $11|(a - b + c - d + e)$.
 - (b) Let n be a positive integer. Prove that $n^{13} - n$ is divisible by 2, 3, 5, 7 and 13.
 - (c) Find the remainder when $35!$ is divided by 37.

