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The Open University of Sri Lanka
B.Sc. Degree Programme
Level 05 – NBT 2017/2018
Pure Mathematics
PEU5300- Riemann Integration

Duration : 1 hour

Date: 10-02-2019 **Time: 1.00pm- 2.00pm**

Answer all Questions

Q1)

Let $f(x) = \begin{cases} 1 & x \in (0,1) \\ 4 & x = 0,1 \end{cases}$ and $g(x) = 2, x \in [0,1]$.

Show that $f(x) \nleq g(x)$ for each $x \in [0,1]$ but $\int_0^1 f(x)dx < \int_0^1 g(x)dx$.

Q2)

Let f be a bounded, increasing function on $[a,b]$. Prove that f is Riemann integrable on $[a,b]$.

Deduce that the function $f(x) = \sin x$ is Riemann integrable on $[0, \pi/2]$. Determine $\int_0^{\pi/2} \sin x \, dx$ by finding the common value of $U(f)$ and $L(f)$.

Q3)

By assuming the theorem that "if $f: [a,b] \to \mathbb{R}$ is Riemann integrable, $g: [c,d] \to \mathbb{R}$ is continuous and $f([a,b]) \subseteq [c,d]$, then the composition $g \circ f: [a,b] \to \mathbb{R}$ is Riemann integrable", deduce that if h is a Riemann integrable on $[a,b]$ then $|h|$ is Riemann integrable on $[a,b]$.

By giving a counterexample show that $|h|$ is Riemann integrable on $[a,b]$ does not necessarily imply that h is Riemann integrable on $[a,b]$.

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