THE OPEN UNIVERSITY OF SRI LANKA

Faculty of Engineering Technology

Department of Mathematics and Philosophy of Engineering
Bachelor of Industrial Studies Honors

Final Examination (2023/2024)

MHZ4357: Applied Statistics

Index No: .....

Date: 20/02/2025 Time: 09:30-12:30 hours

**Instructions:**

- **Part A is Compulsory**
  - Provide short answers in given space.
  - Do not need to show any workings.
- Answer five (05) questions only from Part B.
  - Provide answers in separate sheets (answer booklet) which will be given in the examination.
  - Show all your workings.
- Number of pages in the paper is Eight (08).
- All the symbols are in standard notation unless they are defined.
- All the relevant statistical tables are attached with this paper.
- Do not need to use graph sheets in this paper.
- Non-Programmble Calculators are allowed to used.
- **Attach the Part A to the Answer script of the Part B.**
- This is a closed book test and do not use red color pen.

## Part A

**Provide short answers in the given space. Do not need to show any workings or works. Please attach this part to your answer script.**

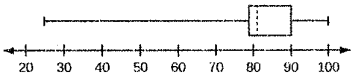
1. Give examples of variables that are on an interval scale and a ratio scale.

   Answer:.....

   ..... *(Marks 10)*

2. What is the most suitable measure of central tendency for the data represented by the following Box and Whisker Plot?

   Answer:.....

   .....

   .....

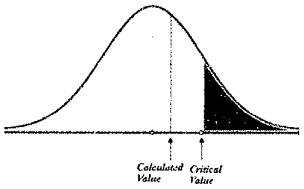
   *(Marks 10)*

3. If 25 is the mean of a random variable $X$ that follows a Binomial distribution with 100 trials, what is the success probability?

   Answer:.....

   *(Marks 10)*

4. What is the decision that can be made according to the position of calculated and critical $Z$ values as displayed in the graph when doing hypothesis testing?




Answer:.....
.....
.....

(*Marks 10*)

5. Consider the scatter plot given in following figure and comment about the relationship between the two variables $x$ and $y$.

Answer:.....
.....
.....

(*Marks 10*)

6. Let $A$ and $B$ be two independent events such that $P(A \cap B) = 2/5$ and $P(B) = 1/2$. Find the values of $P(A \cup B)$.

   Answer:..... (*Marks 10*)

7. Given the observations $x, 5, 6, y, 7, 8$ in ascending order. Suppose that both the mean and the mode of these observations are 6. What are the values of $x$ and $y$?

   Answer:.....

   (*Marks 10*)

8. What is the meaning of the level of significance in hypothesis testing?

   Answer:.....
   ..... (*Marks 10*)

9. Suppose that you want to calculate the 90% confidence interval for the mean value of a normally distributed population with a sample variance $s^2 = 4$ with a sample of 36 units. Find the Magin error of mean.

   Answer:..... (*Marks 10*)

10. A researcher wants to examine the impact of various level of fertilizer on yeild of the fruits (Kg per plant). Identify the response and predictor variables related to the above experiment.

    Answer:.....
    ..... (*Marks 10*)

## Part B

1. The following data were recorded by a rice salesman who imports rice from different countries.

   - Variety
   - Flavor: {Mild, Nutty, Earthy, Creamy}
   - Cost per Kg
   - Rice length: {Short, Medium, Long}
   - Number of suppliers

   (a) Identify the type of data associated with the following variables. *(Marks 15)*

   i. Rice length

   ii. Cost per Kg

   iii. Number of suppliers

   (b) State the scale of measurement of the following variables. *(Marks 15)*

   i. Rice length

   ii. Cost per Kg

   iii. Flavor

   (c) Following barchart shows the average price of Three different fruits ( Apple, Banana and Mango) in a shop in last month and this month.

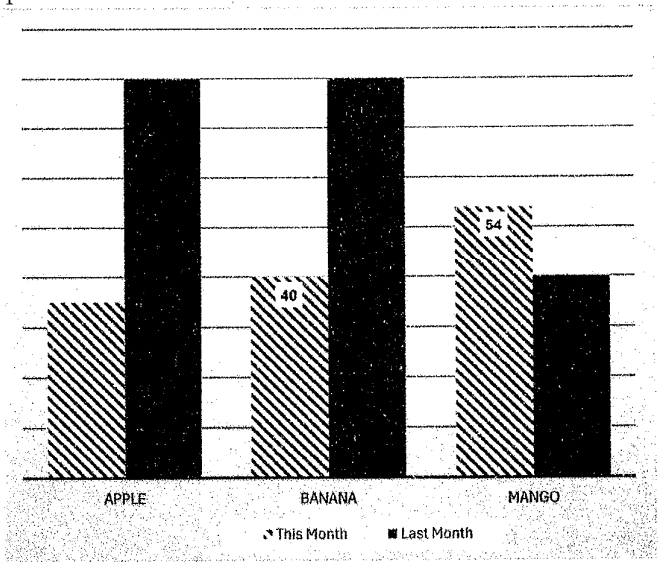



Figure 1:The price of the fruits

i. Given that the price of a Banana in last month is twice as this month. Find the price of a Banana in last month. (*Marks 10*)

ii. If the price of an Apple in this month is Rs.45 cheaper to the last month price, using Figure 1 and the answer in part (i), find the price of an Apple in this month and last month. (*Marks 20*)

iii. Find the price of the mango in last month. (*Marks 10*)

iv. Interprete the Figure 1 by discussing the price changes in each of fruits. (*Marks 10*)

(d) A Greengrocer records his amount of revenue gains from vegiterbles and fruits in last three days ( Friday, Saturday and Sunday)

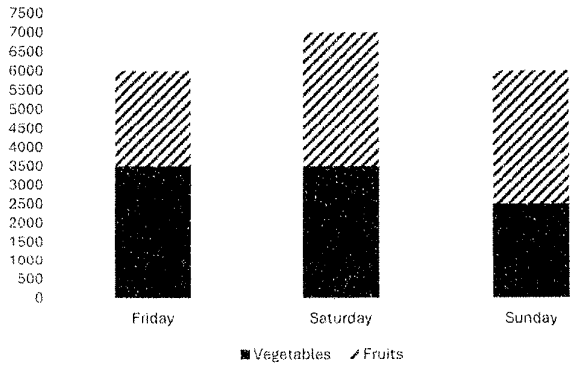



Figure 2:The revenue gains from Vegetables and Fruits

i. Which two days had the same amount of total revenue? (*Marks 05*)

ii. Which two days had the same amount of revenue from fruits? (*Marks 05*)

iii. Interprete the Figure 2 by discussing the revenue gains from vegetables and fruits on given three days. (*Marks 10*)

2. The number of tomato fruits yield by 20 trees were recorded in the following ungroup frequency distribution.

| Number of Fruits | Frequency |
|---|---|
| 5 | 2 |
| 10 | $f_1$ |
| 15 | 9 |
| 20 | $f_2$ |
| 25 | 1 |

The estimated mean number of tomatos per plant is given as 14.

(a) Determine the values of $f_1$ and $f_2$. (*Marks 20*)

(b) Determine the Median and Mode of the above sample. (*Marks 10*)

(c) Find the inter-quartile range for the sample data. (*Marks 20*)

(d) Draw the Box-Whisker plot for the above data. (*Marks 20*)

(e) Are there any outliers in the given data set? Justify your answer. (*Marks 20*)

(f) What are the best suited measures for the central tendency? Explain your answer. (*Marks 10*)

3. A farmer in Sri Lanka has set up pest traps in his field to monitor the number of pests captured per day. Based on historical data, the number of pests captured follows a discrete probability distribution, as shown below:

| Number of Pests ($X$) | Probability $P(X)$ |
|---|---|
| 0 | 0.25 |
| 1 | 0.35 |
| 2 | 0.20 |
| 3 | $p$ |
| 4 | 0.05 |

(a) Find the value of $p$ here. (*Marks 10*)

(b) What is the probability that the farmer will capture at most 2 pests in a day? (*Marks 15*)

(c) Find the probability that the farmer will capture at least one pest and at most 3 pests in a day? (*Marks 25*)

(d) What is the expected number of pests the farmer can expect to capture in a day? (*Marks 25*)

(e) What is the varinace of the number of pests the farmer can expect to capture in a day? (*Marks 25*)

4. (a) Discuss the characteristics of Geometric distribution and the characteristic of Binomial distribution. (*Marks 20*)

(b) A fruit harvester machine is used to detect proper fruits for use. Previous studies confirm that the probability of a fruit being an unusable fruit is 0.02.

i. Find the probability that the first unusable fruit is obtained at the 5th inspected fruit. (*Marks 20*)

ii. What is the probability that the machine will inspect more than 3 fruits to detect the first defective item? (*Marks 20*)

iii. Let the same machine is used to identifyt the unusable fruits in a batch of 20 fruits. Find the probability that 10% of the batch of the fruits were unusable. (*Marks 40*)

5. (a) Define the following terms.

   i. Standard error of the mean (*Marks 10*)

   ii. Magin of error of the mean (*Marks 10*)

   (b) An agricultural researcher plants 36 plots with a new variety of corn. The average yield for these plots is $\bar{x} = 180$ bushels per acre and the standard deviation of the yield in these plots is $s = 10$ bushels.

   i. Give an estimate for the average yield of the new variety of the corn plant. (*Marks 05*)

   ii. Find the standard error of the yield of the new variety of the corn plant. (*Marks 15*)

   iii. Find a 90% confidence interval for the average yield of the new variety of the corn plant. (*Marks 35*)

   iv. Interpret 90% confidence interval in a meaningful way. (*Marks 10*)

   v. Find the probability that true average yield of the new variety of the corn plant is between 176.73 bushels to 183.27 bushels. (*Marks 15*)

6. A fertilizer company claims that using its new organic fertilizer increases the average yield of wheat to more than 50 bushels per acre. To test this claim, an agricultural researcher collects yield data from 40 wheat fields that used the new fertilizer and he found that the average yield is 53 bushels/acre and standard deviation of the sample data is 6 bushels/arce.

   (a) State the corresponding null and alternative hypothesisces that the researcher will test against the fertilizer company's claim. (*Marks 20*)

   (b) Find the test statistics associated with this hypothesis testing. (*Marks 20*)

   (c) If the researcher wants to test this hypothesis at 5% significant level, then what is the critical value/s associates with this test? (*Marks 15*)

   (d) Explain why did you choose the corresponding table when finding critical value/s in part (c). (*Marks 10*)

(e) Test the hypothesis at 5% significant level and state your decision regarding the hypothesis. Explain your conclusion regarding the claim of the customer.

(*Marks 35*)

7. A research institute wants to examine whether the amount of fertilizer used (in kg per acre) has an impact on the yield of corn (in bushels per acre). A sample of 12 farms was taken, and the following data was collected:

| The amount of fertilizer used (in kg per acre) | 5 | 8 | 12 | 6 | 4 | 10 | 15 | 7 | 9 | 3 | 11 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| The yield of corn (in bushels per acre) | 6 | 7 | 8 | 6 | 5 | 9 | 9 | 6 | 8 | 5 | 8 | 9 |

(a) Identify the independent variable and the dependent variable. (*Marks 10*)

(b) Calculate the correlation coefficient for above data. Interpret your answer.

(*Marks 25*)

(c) Draw the scatter plot for dependent variable against the independent variable.

(*Marks 10*)

(d) Find the coefficient of the equation of the best fitted simple linear regression model of the above data. (*Marks 25*)

(e) Interpret the found regression coefficient in part (d). (*Marks 10*)

(f) Use the estimated line of regression to find the yield of corn which was applied 13 kg per acre fertilizers. (*Marks 10*)

(g) State the assumptions that you have to make within this analysis. (*Marks 10*)

End.

**Important Formulars:**

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

Binomial distribution

$$Pr(X = r) = \frac{n!}{(r!(n-r)!)} p^r q^{(n-r)}$$

Geometric distribution

$$Pr(X = r) = pq^{(r-1)}$$

Poison Distribution

$$Pr(X = r) = e^{(-\pi)} \frac{\mu^r}{r!}$$

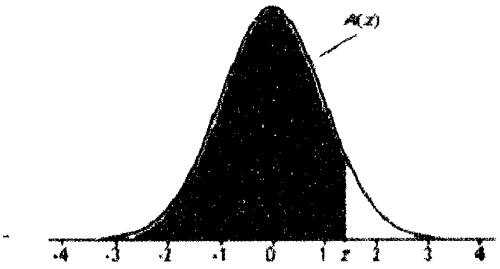
If random variable $X$, follows normal distribution ($X \sim N(\mu, \sigma^2)$) Then $Z = \frac{(X-\mu)}{\sigma}$ follows standard normal distribution where $Z \sim (1, 0)$.

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

$$\beta_1 = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

# Z-table

## Cumulative Standardized Normal Distribution




$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to $z$ (in other words, the area under the curve to the left of $z$). It gives the probability of a normal random variable not being more than $z$ standard deviations above its mean. Values of $z$ of particular importance:

| $z$ | $A(z)$ | |
|---|---|---|
| 1.645 | 0.9500 | Lower limit of right 5% tail |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail |
| 2.326 | 0.9900 | Lower limit of right 1% tail |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | | | | | | | |

## *t* Distribution: Critical Values of *t*

| | | *Significance level* | | | | | |
|---|---|---|---|---|---|---|---|
| *Degrees of freedom* | *Two-tailed test:* | 10% | 5% | 2% | 1% | 0.2% | 0.1% |
| | *One-tailed test:* | 5% | 2.5% | 1% | 0.5% | 0.1% | 0.05% |
| **1** | | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 | 636.619 |
| **2** | | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| **3** | | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| **4** | | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| **5** | | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| **6** | | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| **7** | | 1.894 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| **8** | | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| **9** | | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| **10** | | 1.813 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| **11** | | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| **12** | | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| **13** | | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| **14** | | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| **15** | | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| **16** | | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| **17** | | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| **18** | | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| **19** | | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| **20** | | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| **21** | | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| **22** | | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| **23** | | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| **24** | | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| **25** | | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| **26** | | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| **27** | | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| **28** | | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| **29** | | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| **30** | | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| **32** | | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 |
| **34** | | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 |
| **36** | | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 |
| **38** | | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| **40** | | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| **42** | | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 | 3.538 |
| **44** | | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 | 3.526 |
| **46** | | 1.679 | 2.013 | 2.410 | 2.687 | 3.277 | 3.515 |
| **48** | | 1.677 | 2.011 | 2.407 | 2.682 | 3.269 | 3.505 |
| **50** | | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| **60** | | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| **70** | | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 | 3.435 |
| **80** | | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| **90** | | 1.662 | 1.987 | 2.368 | 2.632 | 3.183 | 3.402 |
| **100** | | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| **120** | | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| **150** | | 1.655 | 1.976 | 2.351 | 2.609 | 3.145 | 3.357 |
| **200** | | 1.653 | 1.972 | 2.345 | 2.601 | 3.131 | 3.340 |
| **300** | | 1.650 | 1.968 | 2.339 | 2.592 | 3.118 | 3.323 |
| **400** | | 1.649 | 1.966 | 2.336 | 2.588 | 3.111 | 3.315 |
| **500** | | 1.648 | 1.965 | 2.334 | 2.586 | 3.107 | 3.310 |
| **600** | | 1.647 | 1.964 | 2.333 | 2.584 | 3.104 | 3.307 |
| **∞** | | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |