THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Industrial Studies Honours

Final Examination (2023/2024)
MHZ5570: Quantitative Techniques

Date: 05th February 2025(Wednesday) **Time: 09:30- 12:30**

Instructions:

- *This paper consists of 6 questions in Five pages.*
- *The first question is compulsory and answer any four (4) other questions.*
- *Relevant equations are provided.*
- *State any assumptions you required and show all your workings.*
- *This is a closed book test and do not use red color pen.*
- *Non-Programmable Calculators are allowed.*

1.

a. Solve for x, $4^{x+2} - 4 = 60$. (Marks 10)

b. Find the value of $\log_5(25)^2$. (Marks 05)

c. Find x such that $\log 4 - \log 8 = \log 20^{(x-1)} - \log 2^x$. (Marks 10)

d. Find the points of intersection of the graph of function $y = x^2 + 3x - 4$ and x axis. (Marks 05)

e. Find the gradient of the curve $y = 2x^5 - \frac{1}{x} + \log_e x$ at $x = 1$. (Marks 15)

f. Let $A \equiv \begin{bmatrix} 4 & 2 & 0 \\ 12 & -3 & 5 \end{bmatrix}$ and $B \equiv \begin{bmatrix} -4 & 10 \\ 5 & -3 \\ -1 & 2 \end{bmatrix}$. Then find the difference of matrices $A^T - B$. (Marks 05)

g. Let $A \equiv \begin{bmatrix} 1 & 3 \\ 5 & 7 \\ -1 & 2 \end{bmatrix}$ and $B \equiv \begin{bmatrix} 4 & -2 \\ 1 & 5 \end{bmatrix}$. Then Find the product of matrices AB.

(Marks 10)

h. Find the inverse of the matrix $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$. (Marks 10)

i. A company uses 7300 units for an item annually, delivery lead time is 8 days. Find the reorder point, in number of units, to order optimum quantity. Assume that a year consists with 365 days. (Marks 10)

j. A company produces two goods, *A* and *B*, for a set price of Rs. 100 for both. One piece of each of *A* and *B* costs Rs. 10 and Rs. 12, to manufacture. The cost of packing each item was an additional Rs. 2. Each item of *A* charges Rs. 14, but each item of *B* charges Rs. 19 for transportation. Write an expression for manufacturer's total profit by clearly stating your variables and assumptions.

(Marks 20)

2. Consider the following systems of linear equations.

$$3x_1 - 3x_2 + 2x_3 = 6$$

$$2x_1 - x_2 - 2x_3 = -8$$
$$5x_1 - 5x_2 + 3x_3 = -8$$

a. Write the system in the matrix form of $AX = H$. Find the determinant of Matrix A.
(Marks 35)

b. Solve the above systems of equations using the inverse of matrix A.

(Marks 65)

3. Let $y = x^3 - 3x^2$, for $x \in \mathbb{R}$.

a. Find $\frac{d^2y}{dx^2}$. (Marks 40)

b. Find minimum/maximum /inflexion points of y. Justify your answer.

(Marks 60)

4. A textile mill is striving to optimize its operations by minimizing material costs while meeting production demands. The mill produces two popular fabrics: Fabric X and Fabric Y, both of which are in high demand. To create these fabrics, the mill relies on three essential raw materials: Cotton, Silk, and Wool. The production process for each fabric has specific material requirements as follows:

- Each unit of Fabric X requires 3g of Cotton, 2g of Silk, and 1g of Wool.
- Each unit of Fabric Y requires 1g of Cotton, 4g of Silk, and 2g of Wool.

Further it is given that the available raw material stokes as 60g from Cotton, 80g from Silk and 50g from Wool. In terms of costs, producing one unit of Fabric X incurs a material cost of Rs. 50, while producing one unit of Fabric Y costs Rs. 60. Currently, the mill needs to meet a regular demand of 15 units of Fabric X and 12 units of Fabric Y. With limited resources and rising material costs, the mill's management is focused on devising a cost-effective production strategy that meets these requirements without exceeding their budget.

a. If linear programming techniques are used to decide how much to produce by minimizing cost, then
   i. identify the decision variables associated with this problem. (Marks 10)
   ii. define the objective function of the linear programming problem. (Marks 15)
   iii. write down the constraints associated with this problem. (Marks 35)

b. Solve this linear program graphically.
   i. Find the feasible region. (Marks 20)
   ii. Identify the optimal solution and the maximum revenue. (Marks 20)

5.

a. Consider the following linear programming problem.

Maximize $Z = 3x_1 + 2x_2$

Subject to:

$$5x_1 + 2x_2 \leq 50 \quad \text{(Fabric availability constraint)}$$
$$3x_1 + 4x_2 \leq 36 \quad \text{(Labor hours constraint)}$$
$$x_1 \geq 8 \quad \text{(Minimum units of casual shirts)}$$
$$x_1, x_2 \geq 0 \quad \text{(Non-negativity constraints)}$$

Where:

x_1: Number of casual shirts produced

x_2: Number of formal shirts produced

   i. By introducing slack or surplus variables, write down the above linear programming problem in standard form. (Marks 20)
   ii. Discuss the usefulness of artificial variables in solving this problem using the simplex method. (Marks 15)

b. The following mathematical model was developed to identify the optimal production which maximizes the profit under the given limitations. Here the company produces two types of products namely P and Q.

$$\text{maximize: } z = 3x + 2y$$

$$\text{subject to:}$$

$$x - y \leq 12$$

$$x \leq 10$$

$$x \geq 0, y \geq 0.$$

In the model, x refers to the number of units of product P and y refers to number of units product Q, produced by the company and z refers total profit gain in production plan of x number of units of product P and y number of units of product Q.
Use simplex algorithm to find the optimal production plan. Discuss whether you are able to find the optimal solution or not. If not explain the reason. If found, state the optimal solution. (Marks 65)

6.

a. Discuss the importance of maintaining inventories in the apparel industry to handle seasonal demand fluctuations and supply chain delays. (Marks 20)

b. A company manufacturing casual shirts has the following data.

- Annual demand for casual shirts: 12,000 units
- Ordering cost per order: Rs.1,500
- Holding cost per unit per year: Rs.20
- Unit cost of each shirt: Rs.500

Using the Economic Order Quantity (EOQ) model,

i. find the optimal number of shirts the company should order per batch to minimize the total inventory cost. (Marks 10)

ii. state the assumptions made while solving part (i), if there are any. (Marks 10)

iii. determine the maximum inventory level the company will maintain at any point. (Marks 10)

iv. assuming that there are 360 operating days in a year, find the optimal cycle time in days. (Marks 10)

v. find the optimal number of orders per year that will minimize the total cost of inventory. (Marks 10)

vi. calculate the total cost of carrying and ordering inventories per cycle when the company uses the optimal order quantity. (Marks 30)

7.

a. Specify the advantages and disadvantages of maintaining inventory in the apparel manufacturing business. (Marks 20)

b. A fashion manufacturer receives a monthly order of 1,200 jackets and has the capability to produce 5,000 jackets per month. The monthly holding cost per jacket in inventory is Rs. 60, and the production (setup) cost per batch is Rs. 2,500.

i. Calculate the optimal batch production size, that minimizes the total inventory cost. (Marks 25)

ii. Determine the number of days when both production and sales will overlap during a month. (Marks 15)

iii. Identify the number of days in a month when only sales will take place. (Marks 10)

iv. How often should production batches be scheduled each month to meet the demand? (Marks 10)

v. Due to limited storage capacity, the maximum inventory size is restricted to 3,000 jackets. Find the new optimal production quantity and the corresponding total cost. (Marks 20)

End

Equations

$$Q^* = \sqrt{\frac{2C_0D}{C_c}}$$

$$T = C_o\frac{D}{Q} + C_c\frac{Q}{2} + C_pQ$$

$$T = C_o\frac{D}{Q} + C_c\frac{Q}{2}$$

$$Q^* = \sqrt{\frac{2C_0D}{C_c}}\sqrt{\frac{C_s + C_c}{C_s}}$$

$$S^* = \sqrt{\frac{2C_0D}{C_s}}\sqrt{\frac{C_c}{C_s + C_c}}$$

$$Q^* - S^* = \sqrt{\frac{2C_0D}{C_s}}\sqrt{\frac{C_c}{C_s + C_c}}$$

$$Q^* = \sqrt{\frac{2C_0r}{C_c(1 - \frac{r}{p})}}$$

$$t_1^* = \frac{Q^*}{p}$$

$$t_2^* = \frac{Q^*}{r}(1 - \frac{r}{p})$$