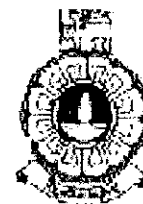**The Open University of Sri Lanka**

B.Sc/B.Ed. Degree Programme – Level 04

Open Book Test (NBT) - 2017/2018

Pure Mathematics

PEU4303 – Group Theory I

Duration: - One Hour.

Date: -09.02.2019   **Time: - 2.30 p.m. – 3.30 p.m.**

Answer All Questions.

01. Let $S = \left\{ a + \sqrt{2}\, b \; : a, b \in \mathbb{Z} \right\}$.

   (a). Show that S is closed under usual multiplication.

   (b). Find the multiplicative identity of S.

   (c). Is S a group under multiplication?. Justify your answer.

   (d). Show that S is a group under usual addition.

02. Let $(G, *)$ be a group.

   (a). Prove that $O(g) = O(g^{-1})$ for each $g \in G$.

   (b). Write down all the elements of $(\mathbb{Z}_8)^{\times}$.

   (c). Find the order of each elements of $(\mathbb{Z}_8)^{\times}$. Justify your answer.

   (d). Find the inverse of the each elements of $(\mathbb{Z}_8)^{\times}$. Justify your answer.