THE OPEN UNIVERSITY OF SRI LANKA 075
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Bachelor of Industrial Studies Honors

Final Examination (2022/2023)
MHZ5570: Quantitative Techniques

Date: 17th February 2024 (Saturday) Time: 13:30-16:30 hours .

Instructions:

- This paper consists of Seven (7) questions in Six (6) pages.
- The first question is compulsory and answer any Four (4) other questions.
- Relevant equations are provided.
- State any assumptions you required and show all your workings.
- This is a closed book test and do not use red color pen.

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1. (a) Solve $4^x = 16^{x-3}$. (*Marks 10*)

   (b) Find x such that $\log(x-2) + \log(x+2) = 0$. (*Marks 10*)

   (c) Find the slope of the straight line $8x + 2y = 5$. (*Marks 10*)

   (d) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$. Then find the matrix $A^T B$. (*Marks 15*)

   (e) Find the determinant of the matrix $\begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$. (*Marks 10*)

(f) A company produces two goods, A and B, for a set price of Rs. 100 for both. One piece of each of A and B costs Rs. 10 and Rs. 12, to manufacture. The cost of packing each item was an additional Rs. 2. Each item of A charges the manufacturer Rs. 14, but each item of B charges Rs. 19. Write an expression for manufacturer's total profit by clearly stating your variables. *(Marks 15)*

(g) Find the minimum value of $Z = x + y$ subjected to constraints $x + 3y \geq 3$, $3x + 2y \leq 6, x, y \geq 0$.

(Marks 20)

(h) The annual usage of a certain item and the delivery lead time are given as 7200 units and 8 days respectively. Find the reorder point, in number of units, to order optimum quantity. Assume that a year consists with 360days.

(Marks 10)

2. Consider the following system of linear equations.

$$\begin{aligned} x_1 - 4x_2 - 2x_3 &= -1 \\ -3x_2 - 2x_3 &= 2 \\ -3x_1 + 4x_2 + x_3 &= 1. \end{aligned}$$

(a) Write the above system in matrix form of $AX = H$. *(Marks 25)*

(b) Solve the above system of equations using the inverse of matrix A. *(Marks 75)*

3. Let $y = 8x^2 - x^4$ on the interval $(-1, 2)$.

(a) Find $\dfrac{d^2y}{dx^2}$. *(Marks 40)*

(b) Find the maximum/minimum points of y if there are any on $(-2, 2)$. Justify your answer.

(Marks 60)

4. A company is involved in the production of two items (X and Y). The resources need to produce X and Y are twofold, namely machine time for automatic processing and man time for hand finishing. The table below gives the number of minutes required for one unit of item:

Item	Machine time (Minutes)	Man time(Minutes)
X	13	20
Y	19	29

The company has 40 hours of machine time available in the next working week but only 35 hours of man time. The revenue received for one unit of item produced (all production is sold) is Rs 200 for X and Rs. 300 for Y. The company has a specific contract to produce 10 units of product X per week for a particular customer.

(a) Let linear programming techniques is used to decide how much to produce per week by maximizing revenue,

i. Identify the desicion variables associate with this problem. (*Marks 10*)

ii. Define the objective function of the linear programming problem. (*Marks 15*)

iii. Wrtite down the constraints associate with this problem. (*Marks 35*)

(b) Solve this linear program graphically. (*Marks 40*)

5. Consider the following linear programming problem.

$$\text{Maximize } Z = 5x_1 - x_2$$

subject to:

$$2x_1 + x_2 = 6$$

$$x_1 + x_2 \leq 4$$

$$x_1 + 2x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

(a) By introducing slack or surplus variables, write down the above linear programming problem in standard form.

(*Marks 20*)

(b) Discuss the neediness of artificial variables in solving this problem with simplex method.

(*Marks 05*)

(c) Write down the standard form of the Linear programming problem for phase I in two phase method by including slack, surplus and artificial variables.

(*Marks 10*)

(d) Find the solution of phase I of the linear programming problem with use of two Phased Simplex method.

(*Marks 40*)

(e) Find the solution of phase II of the linear programming problem with use of two Phased Simplex method.

(*Marks 25*)

6. (a) Discuss the importance of maintaining inventories in the apparel industry.

(*Marks 20*)

(b) The annual demand for a certain frabric product is given as 80000 units. The ordering cost per order is Rs.1200, the cost per unit is Rs 50 and the inventory hodlding cost per unit is given as 6% of the unit cost.

i. Find the optimal number of units which should order per order by the company to minimize the total inventory cost? (*Marks 20*)

ii. State all the assumptions that you made in part (a) if there are any.

(*Marks 10*)

iii. Find the maximum level of inventory. that company will maintain by minimizing inventory cost. (*Marks 10*)

iv. Assuming that there are 360 days in a year, find the optimal cycle time in days. (*Marks 10*)

v. Find the optimal number of of orders per year that should minimize the total cost of inventory. (*Marks 10*)

vi. What will be the total cost of carrying and ordering inventories per cycle when order the optimal order quantity. (*Marks 20*)

7. (a) Specify the advantages and disadvantages of holding inventory in manufacturing business. (*Marks 20*)

(b) A garment manufacturer has an order of 900 T-shirts per month for a customer and it has a capacity to produce 3600 T-shirts per month . The cost of holding a T-shirt in inventory is Rs. 50 per month and the production(set up) cost per production(set up) is Rs. 2000.

i. Find the optimal production quantity of a single production which will minimize the total inventory cost. (*Marks 25*)

ii. Find the period in days where the both production and selling (purchase by school) will be. (*Marks 15*)

iii. Find the period in days where only the selling will be. (*Marks 10*)

iv. How frequently the production run have to be made per month to satisfy the demand? (*Marks 10*)

v. Due to the limited availability of machines, if the amount of production has to be limited to 200 amount of T shirts per run, find the minimum total cost of an inventory. (*Marks 20*)

End.

Equations

$$Q^* = \sqrt{\frac{2C_0D}{C_c}}$$

$$T = C_o\frac{D}{Q} + C_c\frac{Q}{2} + C_pQ$$

$$T = C_o\frac{D}{Q} + C_c\frac{Q}{2}$$

$$Q^* = \sqrt{\frac{2C_0D}{C_c}}\sqrt{\frac{C_s + C_c}{C_s}}$$

$$S^* = \sqrt{\frac{2C_0D}{C_s}}\sqrt{\frac{C_c}{C_s + C_c}}$$

$$Q^* - S^* = \sqrt{\frac{2C_0D}{C_s}}\sqrt{\frac{C_c}{C_s + C_c}}$$

$$Q^* = \sqrt{\frac{2C_0r}{C_c(1 - \frac{r}{p})}}$$

$$t_1^* = \frac{Q^*}{p}$$

$$t_2^* = \frac{Q^*}{r}(1 - \frac{r}{p})$$