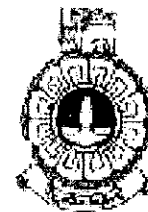


The Open University of Sri Lanka  
B.Sc/B.Ed. Degree Programme – Level 05  
Continuous Assessment Test (OBT) - 2017/2018  
Pure Mathematics  
PEU5303 – Number Theory



Duration: - One Hour.

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Date: -06.01.2019

Time: - 10.30 a.m. -11.30 a.m.

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Answer All Questions.

1. (i) Prove that for any  $n \in \mathbb{N}$ ,  $3 \nmid n^2 + 1$ . (i.e. 3 does not divide  $n^2 + 1$ .)  
(ii) Let  $n \in \mathbb{N}$  and  $n < 1000$ . Prove that  $n$  is either prime or  $n$  must have a prime factor which is less than 37.  
(iii) Let  $M = p^2q^3$ , where  $p, q$  be prime numbers. Write down all the factors of  $M$ .  
(iv) Find  $a, b \in \mathbb{N}$  such that  $a^2 - b^2 \equiv 0 \pmod{11}$  but  $a - b \not\equiv 0 \pmod{11}$ .  
(v) Let  $p$  be prime and  $p > 3$ . Prove that at least one of  $p + 2$  or  $p + 4$  is not prime.
  
2. (i) Find the least element of the set  $\{n : n \in \mathbb{N}, n^2 + n - 110 > 0\}$ .  
(ii) Let  $a, b \in \mathbb{Z}$  and  $a|b$  and  $c|d$ . Prove that  $ac|bd$ .  
(iii) Find the remainder when  $2^{50}$  is divided by 7. Show your work.  
(iv) Using the Euclidean algorithm find the greatest common divisor  $d$  of the numbers 1819 and 3587 and then find integers  $x$  and  $y$  to satisfy  $1819x + 3587y = d$ .

