

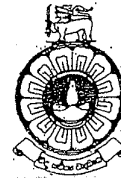
The Open University of Sri Lanka

B.Sc. IT Degree Programme

Final Examination 2024/2025

Level 03 - Applied Mathematics

ADU3330- Discrete Mathematics I



Date: 12/11/2025

Time:09.30am-11.30am

Instructions

- This paper contains two parts of 6 pages (including this page).
- **Part A**-MCQ questions and **Part B**- Essay questions.
- Answer **all** multiple-choice questions (MCQ) in **part A**.
- Answer **three** of five questions in **part B**.
- You are permitted to use non-programmable calculator.
- A total marks obtainable for this examination is 100. The marks assigned for each question and parts thereof are indicated in square brackets.

Part A (Write most correct answer on the answer book.)

01. Suppose p is the statement "I study hard" and q is the statement "I will pass the exam".

Select the correct statement corresponding to symbols $\neg(p \Rightarrow q)$,

- (i) I study hard or I won't pass the exam.
- (ii) I do not study hard, and I will pass the exam.
- (iii) I study hard and I won't pass the exam.
- (iv) I do not study hard and I won't pass the exam.

02. Which of the following proposition is a contradiction?

- (i) $(p \vee q) \wedge \neg p$
- (ii) $(p \Rightarrow q) \wedge (p \wedge \neg q)$
- (iii) $(p \wedge \neg q) \vee q$
- (iv) $(p \Leftrightarrow q) \wedge \neg q$

03. Which of the following proposition is logically equivalent to the compound proposition

$$(\neg p \Rightarrow q) \wedge \neg p ?$$

- (i) $\neg q \vee p$
- (ii) $\neg(\neg q \vee p)$
- (iii) $\neg(q \wedge \neg p)$
- (iv) $\neg p \vee \neg q$

04. Select the statement logically equivalent to “ If the light is not on then there is no electricity”

- (i) The light is on or there is no electricity.
- (ii) The light is not on or there is electricity.
- (iii) The light is on or there is electricity.
- (iv) The light is not on and there is no electricity.

05. Let $A = \{1,2,3,4,5,6,7,8\}$ and $B = \{3,5,7,9,10\}$ be two sets and the set C is defined as $C = A - B$. How many elements in the power set $P(C)$ of the set C ?

- (i) 64 elements.
- (ii) 32 elements.
- (iii) 16 elements.
- (iv) 08 elements.

06. Let $A = \{1,2,3,4\}$ be a given set. The Binary Relation R defined on the set A is given as $R = \{(x, y) | (x, y) \in A \times A, x \text{ is odd and } y \text{ is even}\}$. Then R is,

- (i) $\{(2,1), (3,2), (4,3), (4,4)\}$.
- (ii) $\{(1,2), (1,4), (3,2), (3,4)\}$.
- (iii) $\{(2,1), (2,3), (3,4), (1,2)\}$.
- (iv) $\{(1,3), (2,4), (3,2), (4,1)\}$.

07. Let $P(x)$ be the statement $x^2 + 2 = 11$ and let $A = \{0,1,2,3,4\}$. The quantified statement $\forall x \in A, \neg P(x)$ and its truth value is given by respectively,

- (i) For each x in $A, x^2 + 2 = 11$, “True”.
- (ii) For each x in $A, x^2 + 2 \neq 11$, “True”.
- (iii) For each x in $A, x^2 + 2 \neq 11$, “False”.
- (iv) For each x in $A, x^2 + 2 = 11$, “False”.

08. The function f is defined from A to B where $A = \{1,2,3,4\}$ and $B = \{a,b,c,d,e\}$ with $f(1) = b, f(2) = a, f(3) = d$ and $f(4) = c$. The function f is,

- (i) One-to-one.
- (ii) Onto.
- (iii) Both one-to-one and onto.
- (iv) None of above.

09. Let \mathbb{R} be the set of real numbers. $f: \mathbb{R} \rightarrow \mathbb{R}, g: \mathbb{R} \rightarrow \mathbb{R}$ be two functions defined by

$f(x) = 2x + 5$ and $g(x) = 4x^2 - 1$. Then the function composition $f \circ g(x)$ is given by,

- (i) $6x^2 + 2$.
- (ii) $4x^2 - 3$.
- (iii) $8x^2 + 3$.
- (iv) $8x^2 + 5$.

10. Let $A = \{1,2,3,4\}$ and the relation R is defined as

$R = \{(1,1), (1,2), (2,1), (2,3), (3,3), (4,2), (2,4)\}$. Then R is,

- (i) Reflexive, symmetric and not transitive.
- (ii) Reflexive, not symmetric and not transitive.
- (iii) Not reflexive, symmetric, and transitive.
- (iv) Not reflexive, symmetric and not transitive.

[2.5 × 10 marks]

Part B

01.

- (a) Using truth table, prove that the propositions $(p \wedge q) \vee (\neg p \wedge \neg q)$ and $p \Leftrightarrow q$ are logically equivalent. [10 marks]
- (b) Using laws of logic, determine the compound proposition $[(p \Rightarrow q) \wedge p] \Rightarrow q$ is tautology or a contradiction. [10 marks]
- (c) Let $D = \{-1, 0, 1, 2\}$. Express the statement "There exists x in the domain D such that $x^2 + 10x - 24 = 0$ " using an existential quantifier. Also, determine its truth value. [05 marks]

02. (a) Let x and y be any two elements in the domain A where $A = \{0, 1, 2, 3, 4\}$.Express the quantified statement $\exists y \in A, \forall x \in A, x + y = x$ in words.

Also determine its truth value.

[08 marks]

(b) Let $D = \{-2, -1, 0, 1, 2\}$. Determine the truth value of the statement

$$\forall x \in D, x^2 + 1 \leq 5.$$

[05 marks]

(c) Using the method of contraposition, prove the conditional statement

"If $n^2 + 1$ is even then n is odd" where $n \in \mathbb{Z}$, \mathbb{Z} is the set of integers.

[12 marks]

03.

(a) A coin tossing game is played by two students Amali and Nimali. Each student tosses a fair coin 8 times. In each toss, coin shows either Head(H) or Tail(T). First, one student plays all 8 rounds and receives marks according to the results. Then the second student plays 8 rounds and receives marks in the same way. Following describes how the marks are being assigned.

- For each Head (H) obtained, student receives an **even** numbered mark in the order of the sequence 2,4,6,8,10,12,14,16,18,20.
- For each Tail (T) obtained, student receives an **odd** numbered mark in the order of the sequence 1,3,5,7,9,11,13,15,17,19.
- Each mark can be assigned at most once to a student.
- The range of all possible marks is 1 to 20, which forms the universal set.

*For an example, suppose the sequence of tosses obtained by a particular student is:
H,H,T,H,H,T,T,H.*

Then the marks can be allocated as: 2,4,1,6,8,3,5,10.

Let $A = \{\text{Marks receive by Amali}\}$, $B = \{\text{Marks receive by Nimali}\}$.

Now consider the following sequences of tosses received by Amali and Nimali.

Amali: H,H,T,H,H,H,T,T.

Nimali: T,T,H,T,T,H,T,H.

Based on this:

- (i) Determine the sets A and B .
- (ii) Find $A \cup B$ and $A \cap B$.
- (iii) Find $A - B$ and $B - A$.
- (iv) Let $C = A - B$. Find all the possible subsets of the set C .
- (v) Write the power set of the set C .

[15 marks]

(b) Let $A = \{x|x \in \mathbb{Z}, -2 < x < 2\}$ and $B = \{x|x \in \mathbb{N}, 1 < x < 5\}$ where \mathbb{Z} and \mathbb{N} describe the set of integers and set of natural numbers respectively. Express the sets in element wise form and determine the cartesian products of $A \times B$ and $B \times A$.

[10 marks]

04.

(a) The function f is defined from the set $A = \{\text{Amal, Piyal, Nimal, Kamal, Sunil}\}$ to the set $B = \{\text{January, February, March, April, May, June, July, August}\}$ with $f(\text{Amal}) = \text{March}$, $f(\text{Piyal}) = \text{June}$, $f(\text{Nimal}) = \text{May}$, $f(\text{Kamal}) = \text{February}$ and $f(\text{Sunil}) = \text{March}$.

- (i) State the domain and the range of the function f .
- (ii) What is the codomain of the function f ?
- (iii) Determine whether the function f is one-to-one.
- (iv) Is the function f is onto map? Justify your answer. [08 marks]

(b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x + 3$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $g(x) = x^2 + 3$.

- (i) Find the inverse function $f^{-1}(x)$ of the function $f(x)$. [10 marks]
- (ii) Determine the composite function $f \circ g(x)$. [05 marks]
- (iii) Hence determine the value $f \circ g(1)$. [02 marks]

05.

(a) Let $A = \{1, 2, 2.5, 3\}$ be a given set and let R_1 be a binary relation defined on A where $R_1 = \{(x, y) | (x, y) \in A \times A, x \leq y\}$.

- (i) State the cartesian product $A \times A$. [06 marks]
- (ii) Hence determine the relation R_1 in the element wise form. [05 marks]

(b) Let R_2 be the relation defined on $B = \{p, q, r\}$ is given by $R_2 = \{(p, p), (p, r), (q, r), (r, p), (r, q), (q, q), (r, r)\}$.

- (i) Determine whether the relation R_2 is Reflexive. [03 marks]
- (ii) Determine whether the relation R_2 is Symmetric. [03 marks]
- (iii) Determine whether the relation R_2 is Transitive. [03 marks]
- (iv) Hence deduce whether R_2 is equivalence relation or not. [05 marks]

End of the Question Paper
