

The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

CAT2 - Online Test 2021/2022

Level 03 Pure Mathematics

PEU3202– Vector Spaces

Duration: - One hour

Date: - 29-01-2022

Time: 6.30 p.m. -7.30 p.m.

Answer all questions

1.

- (a) Let M be a vector space over the field F under the usual matrix addition and scalar multiplication.

Let the mapping T be defined by $T(M) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

- (i) Show that T is a linear transformation.
- (ii) Determine whether the following sets are invariant subspaces of the vector space M over the field F under T

I.

II.

(b)

- (i) Define an inner product space.
- (ii) Let V be an inner product space over a field F . Prove that for $u, v \in V$ and $\alpha \in F$

- (iii) let where $\langle u, v \rangle = \alpha \langle u, v \rangle$.

Define $\langle u, v \rangle$. Is $\langle u, v \rangle$ an inner product on V ? Justify your answer.

2.

- (a) Let u and v be any two vectors of a Euclidean Space.

- (i) Prove that $\langle u, v \rangle = \langle v, u \rangle$.
- (ii) Define the angle between u and v .
- (iii) Find the angle between the vectors $(2,2,2)$ and $(0,1,1)$ in E^3 , the usual Euclidean three space.

- (c) Show that the three vectors $u_1 = (2,2,2)$, $u_2 = (0,1,1)$ and $u_3 = (0,0,1)$ form a basis for E^3 , the usual Euclidean three space. Construct an orthogonal basis for E^3 out of $\{u_1, u_2, u_3\}$ using the Gram-Schmidt process.