THE OPEN UNIVERSITY OF SRI LANKA - CITES
Department of Mathematics and Philosophy of Engineering
Bachelor of Software Engineering Honours Study Programme
Final Examination (2020/2021) Batch 1-2
MHZ3459: Basic Mathematics for Computing

Date: 21st August 2022 (Sunday) Time: 10:00 hrs. - 13:00 hrs.

Instructions:

- This paper consists of Five (5) questions in Two (2) pages.
- Answer to all the questions.
- Show all your workings.

1. (a) Let A and B be two square matrices. If the inverse of A^2 is B prove that the inverse of A is AB. [20%]

   (b) Let $A = \begin{bmatrix} 5 & -1 & -1 \\ -5 & 3 & -2 \\ 2 & 0 & 5 \end{bmatrix}$.

   i. For $\lambda \in \mathbb{R}$ define the polynomial $p(\lambda) = \det(A - \lambda I)$, where I is the 3×3 identity matrix. If $\lambda = 2$ is a root of $p(\lambda) = 0$ find other two roots of $p(\lambda) = 0$. [40%]

   ii. If $A^3 - 13A^2 + 52A = 60I$ find A^{-1}. [20%]

   iii. Solve the following system of linear equations by using the method of matrices inverse.

$$\begin{aligned} 5x - y - z &= 3 \\ -5x + 3y - 2z &= -2 \\ 2x + 5z &= 1. \end{aligned}$$

[20%]

2. (a) A company sells 1kg of cement for Rs.120.00 and the daily cost in production of $m$kg of cement $C(m)$, is found as $C(m) = 2m^3 + 124m - 100$.

   i. Show that m satisfies $m^3 + 2m - 50 = 0$ if the company have neither a profit nor a loss in a day. [10%]

   ii. Show that the amount of cement which gives neither a profit nor a loss is between 3 and 4. [10%]

   iii. Use the Newton-Raphson method to find the amount of cement in order to have neither a profit nor a loss to the nearest gram. [30%]

   (b) The cost of producing 3kg, 4kg, and 5kg of cement are Rs.326, Rs.524, and Rs.770 respectively. Use Newton's forward difference interpolations to find the approximation cost for producing 3.5kg of cement to the nearest cent. [50%]

3. Let $f(x) = \dfrac{x^2}{(x-2)^2(x+2)}$ for $x \in \mathbb{R} \setminus \{-2, 2\}$.

   (a) Sketch the graph of $y = f(x)$. Justify all the maxima/minima/inflection points and asymptotes of f if there are any. [50%]

(b) Find the equation of the tangent line at the point $\left(1, \frac{1}{3}\right)$. [25%]

(c) Find the integral $I = \int_0^1 f(x)dx$. [25%]

4. (a) Prove that $\frac{\tan a + \tan b}{\tan a - \tan b} = \frac{\sin(a+b)}{\sin(a-b)}$ and hence find the exact value of $\tan 15°$. [40%]

(b) A survey on a sample of 55 cars being sold at a auto dealer was conducted to see which of three popular options, air-conditioning(A), power windows(P), and fog lights(F) were already installed. The survey found,

32 had air-conditioning
20 had fog lights
11 had air-conditioning and fog lights
6 had all three options
26 had power window
9 had power window and fog lights
13 had air-conditioning and power window

Draw the Venn-diagram to present above data and find the number of cars that had

i. only fog lights
ii. air-conditioning and power window but no fog lights
iii. only one of the options
iv. none of the three options. [60%]

5. (a) Consider the following data regards to the time(in months) that takes to promote to manager from the initial employment of a sample of 100 software engineers from a large telecommunication firm.

Time (months)	10-14	15-19	20-24	25-29	30-34	35-39	40-44
Number of employees	8	12	20	25	22	10	3

i. Find the mean, median, and mode for the given data. [60%]
ii. By constructing a histogram, identify the shape of the distribution. [15%]
iii. What is the best suited measure for the central tendency for the given data? Justify your answer. [5%]

(b) A group of 40 teenagers were asked whether they own an "IPHONE". Their responses are displayed in the following table.

	Boys	Girls	Total
Own an IPHONE	6	4	10
Does not own an IPHONE	18	12	30
Total	24	16	40

Show that if one teenager was selected at random then the following two events are independent. [20%]

B : a boy is selected

I : a IPHONE owner is selected

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