

THE OPEN UNIVERSITY OF SRI LANKA - CITES
 Department of Mathematics and Philosophy of Engineering
 Bachelor of Software Engineering Honours Study Programme



Final Examination 2022/2023 (Batch 4)
 MHZ3459: Basic Mathematics for Computing

Date: 21st October 2023

Time: 09:30 hrs. - 12:30 hrs.

Instructions:

- This paper consists of Five (5) questions in Two (2) pages.
- Answer to ONLY Four(4) questions.
- Show all your workings.

1. (a) Let P, Q, R be three points on $\mathbb{R}^2$ such that $P = (1, 2)$, $Q = (2, 1)$, $R = (3, 3)$. Then find the area of triangle PQR . [25%]

(b) Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 2 & 1 & 3 \end{pmatrix}$.

- Find $\lambda \in \mathbb{R}$ such that $4A^2 - A^3 = \lambda I$, where I is the 3×3 identity matrix. [25%]
- Find A^{-1} (Hint: Use part (i).) [20%]
- Use A^{-1} to solve the following system of linear equations. [30%]

$$x + y = 5$$

$$2x + z = 10$$

$$2x + y + 3z = -5.$$

2. (a) Use the Lagrange's interpolation to find a polynomial y such that

$$y(0) = 0, y(2) = 3, y(4) = 8$$

and hence evaluate y for $x = 3$. [50%]

- (b) Use the method of False Position to find an approximate value for $\sqrt{7}$ correct to two decimal places. (Hint: $2 \leq \sqrt{7} \leq 3$.) [50%]

3. Let $f(x) = \frac{2}{4 - x^2}$ for $x \in D_f$, where D_f is the domain of f .

- (a) Find all the maxima, minima, asymptotes of f if there are any on $\mathbb{R}$. Justify your answer. [40%]

- (b) Evaluate the definite integral $\int_0^1 f(x) dx$. [40%]

- (c) Find the area bounded by the curve $y = f(x)$, $x = 0$, $x = 1$ and, $y = 0$. (Hint: Use part (b).) [20%]

4. (a) Find the value of $a \in \mathbb{R}$ such that the polynomial $p(x) = 4x^3 + 3x^2 - 5x + a$ is divisible by $(x - 2)$. [20%]
- (b) According to a survey with 100 people, 77 of them speak Sinhala, 47 speak Tamil, and 55 speak English. 35 speak Sinhala and English, 30 speak Tamil and English, and 24 speak Tamil as well as Sinhala. Assuming every participants speaks at least one of language answer to the following questions.
- Draw a complete Venn-Diagram according to given above details. [40%]
 - How many of them speak all three languages? [20%]
 - How many of them speak English only? [10%]
 - How many of them speak only Sinhala and Tamil? [10%]
5. (a) (i) The following data represents the monthly earnings (in thousand rupees) of an online publisher who runs a website that publishes information on the basis of monthly payments.

Monthly earnings (in thousand rupees)	Frequency
$60 \leq x < 65$	6
$65 \leq x < 70$	8
$70 \leq x < 75$	5
$75 \leq x < 80$	4
$80 \leq x < 85$	2

- Determine the central tendency (Mode, Median, Mean) for the data. [25%]
 - Draw the histogram for the above frequency distribution. [10%]
 - Draw the frequency polygon for the above distribution on the same diagram of part (ii). [10%]
 - Comment about the shape of the distribution of the data. [05%]
- (b) It is estimated that approximately 40% of emails can be classified as spam emails. Those spam emails were successfully filtered by a software before they reached the recipient's mailbox. The developer of the software says that it can detect 95% of spam emails and has a 4% chance of misclassifying non-spam emails as spam. If the software has marked an email as spam, what is the probability that it is not actually spam? [50%]