

The Open University of Sri Lanka

Faculty of Engineering Technology

Department of Mathematics & Philosophy of Engineering



Study Programme	: Bachelor of Science Honours in Engineering/ Bachelor of Software Engineering Honours
Name of the Examination	: Final Examination
Course Code and Title	: MHZ5355/MHZ5375 Discrete Mathematics
Academic Year	: 2024/25
Date	: 26 th February 2026
Time	: 0930-1230h
Duration	: 3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Six (6)** pages.
3. Answer **Five (5)** out of the **Eight (8)** questions. You must select at least one question from each section (Sections A, B, C, and D).
4. All questions carry equal marks.
5. Answer for each question should commence from a new page.
6. This is a Closed Book Test (CBT).

SECTION – A

Question 01

- (a) Prove the following statements using Mathematical induction.

(i) $7^n - 2n - 1$ is divisible by 4 for all $n \geq 1$;

(ii) $9(2^n) < (n + 3)!$ for all $n \in \mathbb{Z}^+$. (40 marks)

- (b) Let x, y , and z be any integers. Prove that:

(i) If $z \mid x$ and $z \mid y$, then $z \mid (3x + 7y - 2z)$.

(ii) If $x \mid y$ and $y \mid z$, then $|x| = |y|$. (20 marks)

- (c) (i) Find $\gcd(123, 360)$ by applying the Euclid Algorithm.

(ii) By using part (i), find the general solution of $123x + 360y = 99$.

(40 marks)

Question 02

- (a) Let $n > 1$ be fixed and a, b, c, d be arbitrary integers. If $a \equiv b \pmod{n}$, then prove that $a + c \equiv b + c \pmod{n}$ and $ac \equiv bc \pmod{n}$. (20 marks)
- (b) Find all integer solutions to the congruence $42x \equiv 90 \pmod{156}$. (30 marks)
- (c) Find all solutions to the following system of congruences.

$$2x \equiv 3 \pmod{5}$$

$$3x \equiv 2 \pmod{7}$$

$$x \equiv 5 \pmod{6}$$

(50 marks)

SECTION - B**Question 03**

- (a) For each of the following operations " $*$ ", decide whether it defines a binary operation on the given set. Give reasons for your answers.

(i) $x * y = \sqrt{x^2 + y^2}, x, y \in \mathbb{Z}$

(ii) $x * y = \frac{xy}{2}, x, y \in \mathbb{Z}$

(iii) $x * y = \ln(x + y), x, y \in \mathbb{R}^+$

(30 marks)

- (b) Let $A = \mathbb{R} \setminus \{-1\}$ with the operation " $*$ " defined by

$$x * y = x + y + xy$$

- (i) Determine whether the operation " $*$ " is associative.
- (ii) Determine whether the operation " $*$ " is commutative. Justify your answers.

(20 marks)

- (c) Let $G = \mathbb{Z}$ with the operation " $*$ " defined by

$$x * y = x + y - 2$$

- (i) Prove that $(G, *)$ is a group.
- (ii) Determine whether $(G, *)$ is an abelian group. Justify your answer.

(50 marks)

Question 04

- (a) Define a semi-group
- $(G, \#)$
- in usual notation.

Let the operation " $\#$ " : $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$, be defined as follows.

- (i) $x\#y = x + y + 1$, for all $x, y \in \mathbb{R}$
- (ii) $x\#y = \ln(e^x + e^y)$, for all $x, y \in \mathbb{R}$
- (iii) $x\#y = x^2 + y^2$, for all $x, y \in \mathbb{R}$

For each case, determine whether $(\mathbb{R}, \#)$ is a semi-group or not.

Give clear justifications for your answers.

(30 marks)

- (b) Let
- $T = \{1, 3, 7, 9\}$
- be a group under multiplication modulo 10.

- (i) Identify the identity element of T .
- (ii) Find the multiplicative inverse of each element of T modulo 10.

(20 marks)

- (c) Define a group homomorphism using standard notation.

Let $G_1 = (\mathbb{Z}, +)$ be the additive group of integers and

$G_2 = (\{2^n | n \in \mathbb{Z}\}, \times)$ be the multiplicative group of powers of 2.

Define the map $\phi : \mathbb{Z} \rightarrow \{2^n | n \in \mathbb{Z}\}$ defined by $\phi(n) = 2^n, \forall n \in \mathbb{Z}$.

- (i) Show that ϕ is a group homomorphism.
- (ii) Determine whether ϕ is an isomorphism.

Justify your answer.

(50 marks)

SECTION - C**Question 05**

- (a) A feedback system is modeled by the recurrence relation

$y_{n+1} = 3y_n (1 - y_n)$ with the initial value $y_0 = 0.2$.

- (i) Perform 3 iterations to determine y_1, y_2 and y_3 , correct to four decimal places.
- (ii) Based on your results, briefly describe whether the system appears to be converging to a single value or oscillating, as $n \rightarrow \infty$.

(30 marks)

- (b) Consider the complex iterative process defined by the relation:

$$Z_{n+1} = Z_n^2 + c$$

Suppose that the initial value is $Z_0 = (1 - i)$ and the constant is $c = i$.

- (i) Find the values of the iterations from Z_1 up to Z_3 .
 (ii) Discuss the behavior of the sequence Z_n as $n \rightarrow \infty$.

(30 marks)

- (c) Consider a system with two unknowns, x and y , modeled by the following differential equations:

$$\frac{dx}{dt} = -4x + 2y$$

$$\frac{dy}{dt} = -3x + 1y$$

Given initial conditions $x = 2$ and $y = 5$ when $t = 0$.

Find the space values (x_t, y_t) for $t = 1$.

(40 marks)

Question 06

- (a) Let $L = \{a, ab\}$ and $M = \{0, 11\}$ be two languages.

Find the concatenations LM and ML .

(20 marks)

- (b) Let $M = \{S, I, \delta, S_0, F\}$ be a Non-Deterministic Finite Automata (NFA). Where S is a finite set of states, I is a finite set of input symbols, δ is the transition function, S_0 is the initial state, and S_2 is the final state.

Transition Table for the above Non-Deterministic Finite Automata as follows:

States	Input: 0	Input: 1
S_0	$\{S_0, S_1\}$	$\{S_0\}$
S_1	$\{S_2\}$	—
S_2	$\{S_2\}$	$\{S_2\}$

The initial state S_0 , and the final state S_2 .

- (i) Depict the finite automaton's transition graph.
 (ii) Show whether the string 10100 is accepted by applying the transition function.

(50 marks)

- (c) Show that the string $\{a^n c^m d^m b^n \mid n > 0, m > 0, n \neq m\}$ is a sentence generated by the grammar and starting symbol S and production P .

$$P = \{S \rightarrow aSb, S \rightarrow aTb, T \rightarrow cTd, T \rightarrow cd\}.$$

(30 marks)

SECTION – D

Question 07

- (a) By drawing each of the following graphs, determine whether they are simple graphs or not. Give clear reasons for your answer.

(i) $V(G_1) = \{u_i \mid i = 1, 2, 3, 4\}, E(G_1) = \{\{u_i, u_j\} \mid i \neq j\}.$

(ii) $V(G_2) = \{1, 2, 3, 4, 5\}, E(G_2) = \{(x, y), x + y \text{ is odd}\}.$

(30 marks)

- (b) State whether each of the following statements is true or false.

Justify your answer.

- (i) There exists a graph with 8 vertices, 11 edges, four vertices of degree 3 and four vertices of degree 2.

- (ii) If G is a graph with $|V(G)| = 10$, then the maximum possible number of edges in G is 45.

(30 marks)

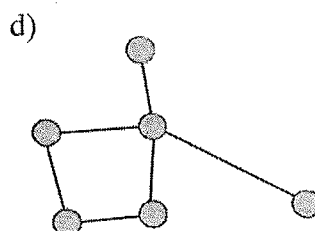
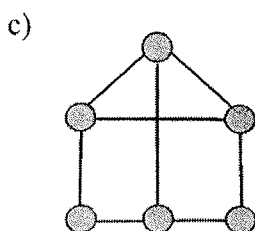
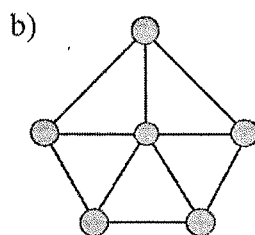
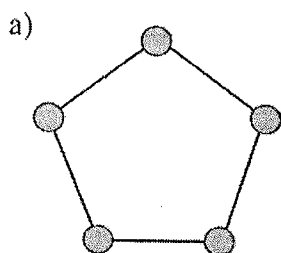
- (c) Let G be a graph with adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

- (i) Without drawing the graph, determine whether G is connected.
- (ii) If $V(G) = \{v_1, v_2, v_3, v_4\}$, find the number of walks of length two (2) from v_1 to v_4 .
- (iii) Draw the graph corresponding to the adjacency matrix A . (40 marks)

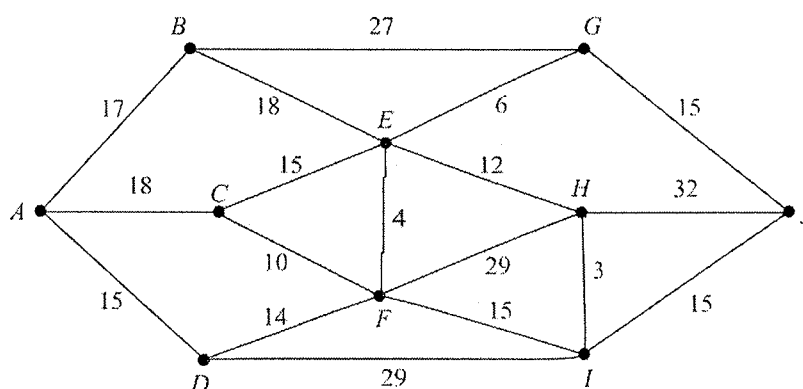
Question 08

- (a) Determine whether the graphs shown below are Eulerian or Hamiltonian. Justify your answers.



(20 marks)

- (b) Use Dijkstra's algorithm to find the shortest route between node A and every other node in the following network.



(60 marks)

- (c) Draw a tree with 10 vertices such that exactly three vertices have degree 3, and all remaining vertices have degree 1 or 2. The tree must satisfy the following conditions:
- Vertices of degree 3 are not adjacent to each other.
 - The tree has exactly five leaves.

Clearly label all vertices and indicate their degrees.
Finally, justify that the graph you have drawn is a tree.

(20 marks)