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The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Study Programme	:	Bachelor of Industrial Studies Honours
Name of the Examination	:	Final Examination
Course Code and Title	:	**MHZ5570 Quantitative Techniques**
Academic Year	:	2024/25
Date	:	27th January 2026
Time	:	0930-1230h
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of Seven (7) questions in (4) pages.
3. The first question is compulsory and answer any Four (4) other questions.
4. Relevant equations are provided.
5. State any assumptions you required and show all your workings.
6. Answer for each question should commence from a new page.
7. This is a Closed Book Test (CBT) and do not use red color pen.
8. Non-programmable calculators are allowed.

Question 01

a. Solve for x, $3^{x-2} + 7 = 250$. (10 marks)

b. Find x such that $\log 3 + \log 5 = \log 45 - \log 9^x$. (10 marks)

c. Find the gradient of straight line joining the points (2,3) and (6,8). (15 Marks)

d. If $A \equiv \begin{bmatrix} -1 & 4 & -2 \\ 3 & 6 & 9 \end{bmatrix}$ and $B \equiv \begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix}$, then find the product of matrices AB. (10 Marks)

e. Find the determinant of the matrix $\begin{bmatrix} 5 & -7 \\ 2 & -3 \end{bmatrix}$. (10 Marks)

f. Calculate the total number of orders placed per year if the Economic Order Quantity is 500 jackets and annual demand is 8,000 jackets. (25 Marks)

g. Calculate the daily consumption rate of blazers given annual demand is 18,000 units and the company operates 300 days/year. (20 Marks)

Question 02

Consider the following systems of linear equations.

$$2x_1 + x_2 - 3x_3 = -4$$

$$4x_1 - x_2 - 4x_3 = 10$$

$$3x_1 - 2x_2 - 2x_3 = -12$$

a. Write the system in the matrix form of $AX = H$. Find the determinant of Matrix A. (35 Marks)

b. Solve the above systems of equations using the inverse of matrix A. (65 Marks)

Question 03

Let $y = 2x^2 - 9x^4$, for $x \in \mathbb{R}$.

a. Find $\frac{d^2y}{dx^2}$. (40 marks)

b. Find minimum/maximum /inflexion points of y. Justify your answer. (60 Marks)

Question 04

A textile finishing unit produces two types of finished fabric rolls: Type Alpha and Type Beta. The production of each roll of Type Alpha requires 5 meters of special fabric and 3 liters of dye, while each roll of Type Beta requires 2 meters of the special fabric and 6 liters of dye. The unit has available 200 meters of special fabric and 240 liters of dye per production cycle. The profit per roll is Rs. 800 for Type Alpha and Rs. 600 for Type Beta.

Management requires that at least 10 rolls of Type Alpha must be produced to fulfill a standing contract, and no more than 30 rolls of Type Beta can be produced due to market limitations.

a. Formulate a linear programming model to determine the number of rolls of each type to produce to maximize total profit. (25 Marks)

b. Solve the problem graphically by identifying the feasible region and all corner points. (40 Marks)

c. Determine the optimal production mix and the corresponding maximum profit. (20 Marks)

d. If the profit per roll of Type Beta increases to Rs. 750, investigate whether the optimal solution changes, without re-solving graphically. (15 Marks)

Question 05

A production company produces two models of handloom machines, Model X and Model Y. Each Model X requires 6 hours of assembly time and 2 hours of testing time, while each Model Y requires 4 hours of assembly time and 4 hours of testing time. The total available assembly time per week is 240 hours, and total testing time is 160 hours.

The profit contribution is Rs. 5,000 per unit of Model X and Rs. 4,000 per unit of Model Y. Additionally, a minimum of 15 units of Model X must be produced each week to meet a government order.

a. Formulate the problem as a linear programming model to maximize weekly profit. (20 Marks)

b. Convert the model into standard form by adding appropriate slack, surplus, and artificial variables. (20 Marks)

c. Construct the initial simplex tableau and perform two complete iterations of the Simplex method. (40 Marks)

d. After two iterations, state whether the current solution is optimal. If not, explain what must be done next and why. (20 Marks)

Question 06

a. What is the main difference between EOQ with shortages and without shortages and list two assumptions of the EOQ model with shortages. (10 Marks)

b. A clothing retailer sells a popular jacket throughout the year. The annual demand for the jacket is 8,000 units. The retailer incurs an ordering cost of Rs. 800 every time an order is placed with the supplier. To store the jackets, the retailer faces a holding cost of Rs. 40 per jacket per year. The retailer allows planned shortages, also known as backorders, when inventory runs out, and estimates that the cost of permitting such shortages is Rs. 100 per jacket per year. The retailer operates for 320 days per year.

Answer the following questions.

i. What is the optimal order quantity that minimizes total inventory costs when planned shortages are permitted? (20 Marks)

ii. Under this ordering policy, what is the maximum level of inventory that will be held at any point during a cycle? (20 Marks)

iii. How many jackets will be backordered at the maximum when shortages are allowed under this optimal plan? (20 Marks)

iv. Once an order is placed and received, how long will it be before the next order is required, expressed in days? (15 Marks)

v. For comparison, compute the Economic Order Quantity if the retailer decides not to permit any shortages at all. (15 Marks)

Question 07

a. What are two advantages and disadvantages of using the Economic Order Quantity (EOQ) model in a manufacturing setting? (10 Marks)

b. A uniform manufacturing company produces blazers at a steady rate of 150 blazers per day to meet an annual demand of 18,000 blazers. The company operates 300 days per year. Each time the production line is set up to begin a batch, a setup cost of Rs. 2,500 is incurred. Additionally, the cost of holding one blazer in inventory for a year is Rs. 50.

Answer the following questions.

i. Determine the optimal production batch size that minimizes the total annual inventory and setup costs, assuming no restrictions on storage capacity. (20 Marks)

ii. Once production of a batch begins, how many days will the company continue to produce blazers before stopping production? (20 Marks)

iii. After production stops, how many days will it take for the accumulated inventory to be completely sold or consumed? (20 Marks)

iv. What is the total cycle time, in days, from the start of one production run to the start of the next? (15 Marks)

v. Under the optimal batch size, how many production runs are required per year, and what is the duration, in days, of each production period? (15 Marks)

Equations:

$$Q^* = \sqrt{\frac{2C_0D}{C_c}}$$

$$T = C_0\frac{D}{Q} + C_c\frac{Q}{2} + C_pQ$$

$$T = C_0\frac{D}{Q} + C_c\frac{Q}{2}$$

$$Q^* = \sqrt{\frac{2C_0D}{C_c}}\sqrt{\frac{C_s + C_c}{C_s}}$$

$$S^* = \sqrt{\frac{2C_0D}{C_s}}\sqrt{\frac{C_c}{C_s + C_c}}$$

$$Q^* - S^* = \sqrt{\frac{2C_0D}{C_s}}\sqrt{\frac{C_c}{C_s + C_c}}$$

$$Q^* = \sqrt{\frac{2C_0r}{C_c(1 - \frac{r}{p})}}$$

$$t^*_1 = \frac{Q^*}{p}$$

$$t^*_2 = \frac{Q^*}{r}(1 - \frac{r}{p})$$