The Open University of Sri Lanka
B.Sc. Degree Programme
Pure Mathematics - Level 04
Open Book Test-2017/2018
PEU4301/PUE4301 — Real Analysis II

DURATION: ONE HOUR

Date: 06. 01. 2019   Time: 2.30 p.m. –3.30 p.m.

ANSWER ALL QUESTIONS.

1. (i) Let $f: [-1, 1] \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 2x^2 + 1 & x \in (0, 2] \\ 2018 & x = 0 \\ 2x - 1 & x \in [-2, 0) \end{cases}$$

Prove that $\lim_{x \to 0^-} f(x) = -1$ and $\lim_{x \to 0^+} f(x) = 1$.

(ii) Let $g(x) = \frac{3x-1}{8x+7}$ for each $x > 0$. Prove that $\lim_{x \to +\infty} g(x) = \frac{3}{8}$.

2.   Let $h: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$h(x) = \begin{cases} 2x^2 + x & \text{if } x \geq 1 \\ 4x - 1 & \text{if } x \leq 1 \end{cases}.$$

By considering the cases $x < 1, x = 1$ and $x > 1$ separately show that h is continuous on $\mathbb{R}$.

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