

The Open University of Sri Lanka
 B.Sc. Degree Programme, Level – 04
 Final Examination – 2025/2026
 PHU4303 – Mathematical Methods for Physics
 Duration: 2 hours



Date: 30th December 2025

Time: 1.30 p.m. to 3.30 p.m.

This paper has 3 pages and 6 questions. Answer any four (4) questions
 Non-programmable calculators are allowed.

1)

a) $A = \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix}$

i) Calculate the Eigen value of matrix A. (15 marks)

ii) Calculate the Eigen vectors of matrix A. (15 marks)

b) The spin of an electron is a quantum property that can be measured along any axis. When measured along the z-axis, we get two possible results: "spin up" or "spin down". These can be 'represented' using the following matrix formats.

$$\text{Spin up} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and Spin down} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

General spin state is given by

$$\text{General spin} = C_1 \times \text{Spin up} + C_2 \times \text{spin down}$$

For a certain electron $C_1 = \frac{\sqrt{3}}{2}$ and $C_2 = \frac{1}{2}$. Calculate the general spin of this electron.

(20 marks)

2)

a) A weather model prediction the air pressure (P) and wind velocity (v) over a region as

$P(x, y) = 1000 - 0.2x - 0.1y$ and Wind velocity in the region as

$$v(x, y) = (10 + 0.02x)\mathbf{i} + (5 - 0.01y)\mathbf{j}$$

where: x = eastward distance in km (0 to 100 km)

and y = northward distance in km (0 to 100 km).

i) Calculate the pressure gradient. (10 marks)

ii) Calculate the divergence of the wind in the region. Is the air moving into the region or moving out of it? (10 marks)

iii) Calculate the curl of the wind. Is there a rotation in the wind or is it moving straight? (10 marks)

iv) What is the fastest wind speed you can expect in the given region? (05 marks)

- b) A research group reports progress of its AI model every three months. It is predicted; IQ of the model will increase by 20% every three months. Starting IQ of the model is 50.

- i) What is its IQ level at the end of first three months? (05 marks)
- ii) What is its IQ level at the end of a year? (05 marks)
- iii) How long does it take to surpass an IQ value of 160? (05 marks)

3)

- a) A cylindrical soda can (with top and bottom covered) must hold 355 cm^3 .

- i) Write an expression for the volume of the can. (10 marks)
- ii) Write an expression for the outside surface area (materials used). (10 marks)
- iii) What dimensions (radius and height) will minimize the amount of material needed to make the can? (10 marks)

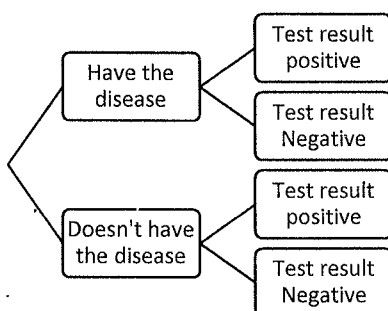
- b) The temperature T in the atmosphere depends on position (x, y, z) and time t . A temperature field is defined as $T(x, y, z, t) = 20 + 0.1x - 0.2y - 0.05z + 0.01t$

- i) Calculate the partial derivate of temperature with respect to time. (10 marks)
- ii) Calculate the total derivative. (10 marks)

4)

- a) A certain disease affects 1 in 1,000 people in a population. A test for the disease has the following properties: If you have the disease, the test will be positive 99% of the time. If you don't have the disease, the test will be negative 95% of the time.

- i) Copy the following diagram to your answer script and fill in the probability values at appropriate branches. (10 marks)



- ii) If 100,000 people were randomly selected and tested, how many tests are expected to be positive. (10 marks)
- iii) If a randomly selected person tests positive, what is the probability he/she has the disease? (10 marks)

b)

i) Calculate $\frac{3+5i}{2-5i}$ where $i = \sqrt{-1}$. (10 marks)

ii) Plot your answer in an Argand diagram. (10 marks)

5)

a) Solve: $\int x^2 \sin(x) \cdot dx$ (20 marks)

b) A function is described as $f(x) = x^2$; $0 \leq x \leq 2\pi$

i) Write the Fourier series equation. (15 marks)

ii) Calculate the Fourier series for above function. (15 marks)

6)

a) Use bi-section method to calculate a numerical solution for $f(x)$, where

$$f(x) : x^5 + 2x - 2 = 0$$

Copy the following table to your answer script to show intermediate steps. Initial values for a and b are -1 and +1. Calculate up to 5 iterations. Use a minimum of 4 decimal points for each calculation. (20 marks)

Iteration	a	b	midpoint	$f(\text{midpoint})$
1	-1	+1		
2				
3				
4				
5				

b) A 2 kg mass (m) is attached to a spring with spring constant $k = 18 \text{ Nm}^{-1}$. The system has a damping force (c) equal to 6 times the velocity.

From Newton's Second Law: $mx'' = -kx - cx'$

where x is the distance from equilibrium and, x'' and x' are the first and second derivatives of x with respect to time.

i) Rewrite the equation using provided values. (10 marks)

ii) Write the characteristic equation. (10 marks)

iii) Find a general solution for the system. (10 marks)

*Allocated marks per question is 50. Final marks for the paper will be calculated by dividing the total marks by two.

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