THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF SOFTWARE ENGINEERING
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

ECZ3161 – MATHEMATICS FOR COMPUTING
FINAL EXAMINATION – 2011/12 **CLOSE BOOK**

Date: February 22, 2012 **Time: 9.30 – 12.30hrs**

Instructions

1. Answer any **five** out of eight questions.
2. Show all steps clearly.
3. **Programmable** calculators are **not** allowed.

Q1

(a) Use Boolean algebra to simplify following expressions.

i) $\overline{abc}\,\overline{d} + ab\bar{c}d + abcd + \overline{abc}\,\overline{d} = \overline{\bar{a}b + a\bar{d} + \bar{b}d}$

ii) $\overline{\overline{abcd} + \bar{a}b\bar{c}d + abcd + a\bar{b}c\bar{d}} = \bar{a}c + a\bar{c} + \bar{b}d + b\bar{d}$

(c) Use Truth tables to show the followings.

i) $x\bar{z} + \bar{y}z + xy\bar{z} = x\bar{z} + \bar{y}z$

ii) $\bar{a}b\bar{c} + a\overline{bc} + \overline{ab}c + abc = \overline{\bar{a}bc + a\bar{b}c + \overline{abc} + ab\bar{c}}$

(c) Use Karnaugh map and find minimal sum for the followings.

i) $xyz + x\bar{y}z + \bar{x}yz + \overline{xy}z$

ii) $xyz\bar{t} + x\bar{y}z\bar{t} + x\overline{yzt} + x\overline{yz}t + \overline{xy}zt + \overline{xy}z\bar{t} + \overline{xyzt} + \bar{x}y\overline{zt}$

Q2

(a)

If $A = \begin{pmatrix} 2 & 0 \\ 3 & -5 \end{pmatrix}$, show that $A^2 + 3A = 10I$; where I is the identity matrix of order 2.

(b)

Let $A = \begin{pmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{pmatrix}$, show that $A^2 = A$

Hence deduce that $(I - A)^2 = (I - A)$, where I is the identity matrix of order 3.

(c) If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, find out values of α, β such that $(\alpha I + \beta A)^2 = A^2$, where I is the identity matrix of order 2.

Q3 Consider 3×3 matrix A,

$$A = \begin{pmatrix} 3 & 2 & 0 \\ -3 & -3 & -1 \\ 4 & 4 & 1 \end{pmatrix}$$

(a) Find the transpose of A

(b) Find the inverse of the transpose of A using Gaussian elimination method.

Q4

(a) Given that $\tan\alpha = \frac{1}{2}$, α in quadrant III, and $\sin\beta = \frac{3}{4}$, β in quadrant II. Find

i) $\sin 2\alpha$ ii) $\cos(\alpha+\beta)$ iii) $\tan(\alpha-\beta)$

Give exact answers and show all your work.

(b) Sketch the graph of $y = \sin^2 x$ in the period $-2\pi \le x \le 2\pi$.

(c) Answer the following problems

i) Find the height of a chimney when it is found that, on walking towards it 50m on a horizontal line through its base, the angular elevation of its top changes from 30° to 45°.

ii) The angle of elevation of the top of an unfinished tower at a point distance 120m from its base is 30°. Find the height of the tower that needs to be raised, when the angle of the elevation is 60° at the same point.

Q5

(a) Let $a = \operatorname{cosec}\theta - \cot\theta$, where θ is not an even multiple of π and $a \neq 0$.

Show that, $\operatorname{cosec}\theta + \cot\theta = \frac{1}{a}$

Deduce that, $\sin\theta = \frac{2a}{1+a^2}$ and $\cos\theta = \pm\frac{1-a^2}{1+a^2}$

(b) Prove the following.

i) $\sin(A+B)+\sin(A-B) = 2\sin A\cos B$

ii) $\frac{2\cos^2 x}{2\cot x - \sin 2x} = \tan x$

iii) $\sin 75^\circ + \sin 15^\circ = \sqrt{\frac{3}{2}}$

(c) If $0^0 \le \theta \le 360^0$ and $\sin 2\theta - \sin\theta = \cos\theta - \cos 2\theta$, then Show that $\tan\frac{3\theta}{2} = 1$.

Hence find the solution of $\tan\frac{3\theta}{2} = 1$, in the range $0^0 \le \theta \le 360^0$.

Q6

(a) Find the following limits

i) $\lim_{x \to 0} \frac{\tan x - x}{\sin x}$

ii) $\lim_{x \to 5} \frac{\sqrt{x-1} - 2}{x^2 - 25}$

iii) $\lim_{x \to 0} \frac{(x+1)^2 - 1}{x(x+1)}$

(b) i) Taking $x_0 = 1$ as an initial approximation for the root of Newton-Raphson formula, Obtain two further approximations for the positive root of $x^2 - 3 = 0$. The answers should be in at least three decimal places.

ii) Given $3x^4 - 8x^3 + 6x^2 = 0.95$, find positive value of x with two iterations correct to two decimal places using Newton-Raphson method starting with $x = 0.8$

Q7

(a) Find first derivatives of the following from first principles. Show all steps.

i) $\frac{1}{x}$ ii) $\cos x$

(b) Find $\frac{dy}{dx}$ as a function of x for

i) $y = \left(x^2 + \sqrt{1 + x^4}\right)^{20}$ ii) $y = x^2 + 3x.\sin 2x.\cos 2x$

(c) If $y = 3x + \frac{1}{2}\sin 2x - 4\sin x$, show that

$$\frac{dy}{dx} = 2(\cos x - 1)^2$$

Q8

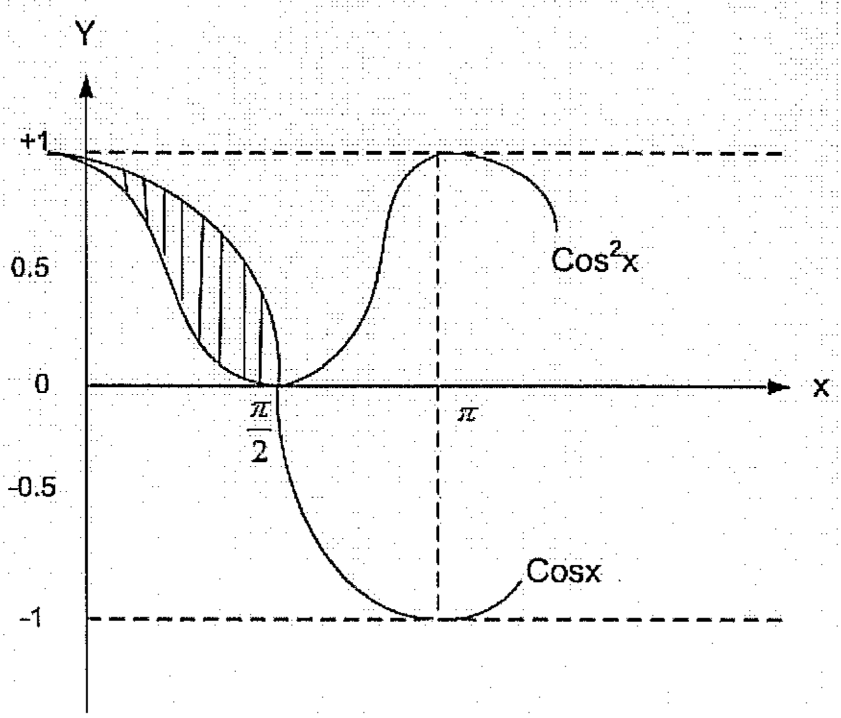
(a) Evaluate the following.

i) $\int(\sin 2x - \cos 3x)dx$ ii) $\int 3\sin(4-5x)dx$

(b) Find the exact value of the following.

i) $\int_0^2 \frac{1}{x^2+9}dx$ ii) $\int_0^2 \frac{1}{\sqrt{4-x^2}}dx$

(c) Consider the following figure with two curves.



i) Write an equation to find the shaded area of the figure by integration method.

ii) Hence, find the shaded area of the figure.

END.