00054

THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY /BACHELOR OF SOFTWARE ENGINEERING – LEVEL 05
FINAL EXAMINATION– 2012/2013
MPZ5140/MPZ5160– DISCRETE MATHEMATICS - II
DURATION – THREE (03) HOURS

Date: 21st July 2013 Time: 930-1230 hours

Instructions:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each section.**
- **State any assumptions that you made.**
- **All symbols are in standard notation.**

SECTION – A

01. i. Define an 'abelian group' by using usual notations.

(20 marks)

ii. Define the commutativity and the associativitity of each of the following binary operations.

a) "$*$" defined on $\mathbb{R}$ by $x * y = y$

b) "$\oplus$" defined on $\mathbb{R}$ by $a \oplus b = (ab)^{1/2}$

(30 marks)

iii. Let G be the sub set of $\mathbb{Z}$. Prove that G is an abelian group with respect to the binary operation "$*$" defined by $a * b = a + b + 3$

(50 marks)

02. i. Let $(G, *)$ be a group. Show that every $a \in G$ has a unique inverse, a^{-1} in G.

(25 marks)

ii. Let G be the set of all real matrices of the form $\begin{pmatrix} a & b \\ o & d \end{pmatrix}$ and $ad \neq 0$.

Prove that G forms a group under matrix multiplication. Verify whether G is an abelian or not?

(75 marks)

03. i. Let G be a group such that $(ab)^2 = a^2b^2$ for all $a,b \in G$. Show that G is an abelian group.

(20 marks)

ii. (a) What is meant by "Homomorphism" used in group theory.

(10 marks)

(b) Let G be a group of real numbers under addition, and let G^* be a group of positive real numbers under multiplication. Show that the mapping: $h: G \to G^*$, defined by $h(a) = e^{2a}$, is a homomorphism.

(30 marks)

iii. Let $G_1 = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \mid a,b \in \mathbb{R} \text{ and } a^2+b^2 \neq 0 \right\}$ and G_2 be the set of all non zero real numbers. Assuming G_1 and G_2 are groups under matrix multiplication and under multiplication respectively. Show that the mapping $\phi: G_1 \to G_2$ *given by* $\phi\begin{pmatrix} a & b \\ -b & a \end{pmatrix} = a^2+b^2$ for all $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \in G_1$ is a homomorphism.

(40 marks)

SECTION – B

04. i. By drawing each of the following graphs, indicate which are simple graphs or not.

a) $G_1 = \{V_1, E_1\}$ *where* $V_1 = \{1,2,3,4,5,6\}$ and
$E_1 = \{\{x,y\} \mid (3x+2y) \text{ is odd and } x \geq y\}$

b) $G_2 = \{V_2, E_2\}$ where $V_2 = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}\}$ and
$E_2 = \{\{n_1,n_2\}, \{n_2,n_3\}, \{n_3,n_5\}, \{n_4,n_5\}, \{n_1,n_3\}, \{n_1,n_6\},$
$\{n_7,n_{10}\}, \{n_6,n_9\}, \{n_1,n_9\}, \{n_5,n_6\}, \{n_8,n_9\}\}$

c) Let G_3 be a graph with $V(G_3) = \{1,2,\ldots\ldots..,9,10\}$, such that two numbers "i" and "j" in $V(G_3)$ are adjacent if and only if $i \times j$ is a multiple of 10. Draw the graph G_3 and determine $E(G_3)$.

(40 marks)

ii. Find the vertices n such that the complete graph, has at least 1200 edges.

(25 marks)

iii. Mr. and Mrs. Chandana gave a party attended by four other couples. Some pairs of people shook hands when they met, but naturally no couple shook hands with each other. At the end of the party, Mr. Chandana asked the other nine people how many hand shakes they had made, and received nine different answers. Since the maximum number of hand shakes any person could make was eight, the nine answers were 0,1,2,3,4,5,6,7,8. Let the vertices of a graph be 0,1,2,3,4,5,6,7,8,C, where C represents Mr. Chandana and the nine numbers represent the other nine people, with person i having made j hand shakes such that $i = j$, *where* $i \in \{0,1,2,3,4,5,6,7,8\}$ *and* $j \in \{0,1,2,3,4,5,6,7,8\}$.

a) Draw the graph representing the above relationships.

b) How many handshakes did Mr. Chandana make?

(35 marks)

05. i. Briefly explain connected graph, multigraph and sub graph.

(15 marks)

ii. Let G be a graph of 14 vertices and 30 edges in which every vertex is of degree 4 or 5. How many vertices of degree 5 does G have? Construct one such graph G.

(30 marks)

iii. Let G_1 be a graph of 8 vertices and 13 edges. Denote by x,y,z the number of vertices in G_1 of degree 2, 3 and 4 respectively. Assume that $y \geq 1$. Find all possible answers for (x,y,z). For each your answer, construct a corresponding graph. (The minimum degree of vertices in graph is 2).

(55 marks)

06. i. G is the graph whose adjacency matrix M is given by

$$M = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

a) If $V(G) = \{n_1, n_2, n_3, n_4\}$, find the number of path of length four (4) joining vertices n_3 and n_4. Give the list of paths if any.

(40 marks)

b) Without drawing a diagram of G, determine whether G is connected or not.

(15 marks)

c) Draw the graph of adjacency matrix M.

(10 marks)

ii. Define a "Tree" graph

(10 marks)

a) Draw a tree with 10 vertices, exactly 05 of which have degree of one.

(10 marks)

b) Show that G is a tree if and only if G has exactly 1 simple path between any 2 vertices.

(15 marks)

SECTION - C

07. i. Draw the graph for the relationship: $x_{n+1} - \lambda x_n(1-x_n) = 0,\ where\ \lambda = 0.6$ and $x_0 = 0.6$.(at least 6 iteration steps are necessary)

(30 marks)

ii. Draw the graph for the relation: $z_{n+1} = z_n^{\ 2} + \lambda,\ where\ \lambda = 0$ and $z_0 = 1.8 + i0.4$. Find z_5 and hence, deduce z_n as $n \to \infty$.

(30 marks)

iii. For the given relationship: $x_{n+1} = \lambda x_n(1-x_n)\ where\ \lambda = 1.6$, obtain the convergent value for each of the following case:

a) $x_0 = 0.3$ b) $x_0 = 0.6$

(40 marks)

08. A three dimensional system is governed by following three differential equations.

$$\frac{dx}{dt} - x - 3y - 3z = 0$$

$$\frac{dy}{dt} + 3x + 5y + 3z = 0$$

$$\frac{dz}{dt} - 3x - 3y - z = 0$$

At $t = 0$, (x,y,z)=(1,1,1). Find the phase space values (x_t, y_t, z_t) for $t=1,2,3$.

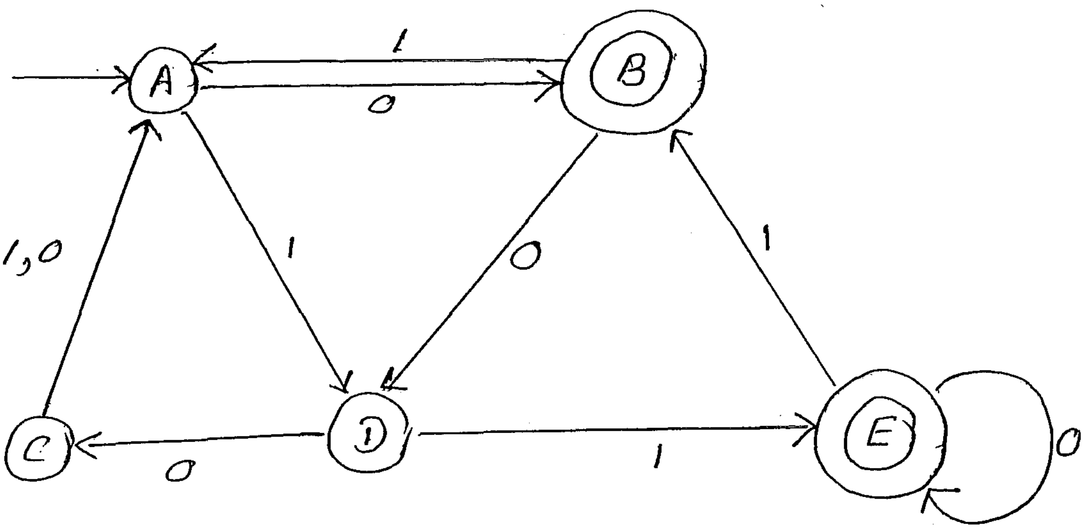
(100 marks)

09. i. Consider the language $L_1 = \{a, ab, ac\}$ and $L_2 = \{a, b, c\}$ over set $A = \{a, b, c\}$. Find $L_1L_2,\ and\ L_1^3$.

(25 marks)

ii. Define a Deterministic Finit Automation (DFA).

Construct the state transition table for the following directed graph of DFA. What are the initial and accepting state.



(25 marks)

iii. Let M be a Mealy machine. Let $s \in S,\ a \in I\ and\ x \in I^*$ and define function $\delta : S \times I^* \to S$ and $\beta^* : S \times I^* \to 0^*$ by

$$\delta^*(s, \Omega) = s$$
$$\delta^*(s, a\ .\ x') = \delta^*[\delta(s, a),\ x']$$
$$\beta^*(s, \Omega) = \Omega$$
$$\beta^*(s,\ a.x') = \beta(s,\ a)\beta^*[\delta(s,\ a),\ x']$$

The two frame binary pipe line device hold up following table.

Status	Input		Output	
	0	1	0	1
000	010	001	0	0
001	010	011	1	0
010	110	101	0	1
011	000	110	1	1
100	001	010	1	0
101	010	000	1	0
110	110	100	1	1
111	111	001	0	1

Find the two frames binary pipe line buffer, and work out its response to the sequence 101001 from state 101.

(50 marks)