THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed. Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS – LEVEL 05
ADU5305/ Statistical Inference
Open Book Test - 2017/2018

Duration: One Hour.

| Date: 29-12-2018 | Time: 9.00 a.m. to 10.00 a.m. |

Non programmable calculators are permitted. Statistical tables are provided.

Answer all questions.

1.

Let X_1, X_2, X_3,......... X_n be a random sample from a distribution with density given by $f(x; \theta)$.   Let $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4$, are functions of X_1, X_2, X_3,......... X_n. Suppose $\hat{\theta}_1, \hat{\theta}_2$ are consistent estimators for parameter θ, $\hat{\theta}_3$ unbiased estimator for parameter θ, $\hat{\theta}_4$, is the likelihood estimator for parameter θ and for large samples $MSE(\ \hat{\theta}_2\ ) < MSE(\ \hat{\theta}_3\ )$. State whether the following statements are true or false. Justify your answer.

  (i)   Always $E(\frac{\hat{\theta}_1 + \hat{\theta}_2}{2}) = \theta$

  (ii)   For large samples $Var(\hat{\theta}_2) < Var(\hat{\theta}_3)$

  (iii) Let $L(\theta)$ be the likelihood function of θ. $L(\hat{\theta}_4) < L(\hat{\theta}_1)$.

  (iv) $\hat{\theta}_3$ is not an accurate estimator for θ

  (v)   For large samples $\hat{\theta}_2$ and $\hat{\theta}_4$ are accurate and precise estimators for θ.

(50 marks)

2.

A company that produces a certain electrical product claims that the life time X (in years) has the density function

$$f(x, \beta) = \beta exp(-\beta x) \qquad ; \ \beta > 0, \ \ x > 0$$

and the moment generating function of X is given by

$$M_X(t) = \frac{\beta}{\beta - t} ; \ \ t < \beta$$

Let $X_1, X_2, ...X_n$ denote lifetimes of n randomly chosen products from the above population.

(i)   Show that the expected life time and variance of a randomly selected electrical product are β^{-1} and β^{-2} respectively.

(ii)   Derive Maximum likelihood estimator for expected life time of a randomly selected electrical product. Is the estimator derived by you an unbiased estimator for expected life time of a randomly selected electrical product? Justify your answer.

(iii)   Derive Maximum likelihood estimator for variance of a randomly selected electrical product.

(iv)   A sample drawn from the above distribution is given below. Estimate the expected life time and variance of a randomly selected electrical product using part (ii) and part (iii).

2.29   8.65   6.74   8.19   3.77   6.27   4.29   15.72   10.78   6.50 0.48 0.87
1.90   2.04   11.97 5.89   30.70 0.03   13.44 21.84

(v)   Find the sample size necessary to estimate the expected life time of a randomly selected electrical product within error bound on six months with 95% confidence.

(50 marks)