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THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. /B.Ed. DEGREE PROGRAMME

APPLIED MATHEMATICS-LEVEL 05

ADU5304 – OPERATIONAL RESEARCH

NO BOOK TEST 2017/2018

Duration: One Hour

Date: 10.02.2019	**Time: 10.30 a.m- 11.30 a.m**

Answer all questions

(1) A dress shop has 3 sales persons. Assume that arrivals follow Poisson pattern with an average of 10 minutes between arrivals. Also assume that any sales person can provide the desired service for any customer. If the time to provide service for a customer is exponentially distributed with a mean of 20 minutes per customer,

(i) what is the probability that no customer in the system?
(ii) what is the probability that one customer in the system?
(iii) what is the average number of customers waiting to be served?
(iv) find the average time that a customer spends in the system.

(2) Customers arrive at one window drive-in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponentially distributed with mean 5 minutes. The space in front of the window including that for the serviced car can accommodate a maximum of 3 cars and other can wait outside this space. Find

(i) the probability that an arriving customer can drive directly to the space in front of the window.
(ii) the probability that an arriving customer will have to wait outside the indicated space
(iii) how long the arriving customer is expected to wait before starting service.

<u>Formulas (in the usual notation)</u>

<u>(M/M/1): (N/FIFO) Queueing System</u>

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \neq 1 \\ \dfrac{1}{N+1}, & \rho = 1 \end{cases} \qquad E(m) = \frac{\rho^2\left[1 - N\rho^{N-1} + (N-1)\rho^N\right]}{(1-\rho)\left(1-\rho^{N+1}\right)}$$

$$E(v) = {[E(n)]}\big/{\lambda'}, \quad \text{where } \lambda' = \lambda(1-P_N)$$

$$E(n)=\frac{\rho\left[1-(N+1)\rho^{N}+N\rho^{N+1}\right]}{(1-\rho)\left(1-\rho^{N+1}\right)}$$

$$E(w) = \frac{1}{\mu}\,(E(n)$$

(M/M/C):(∞/FIFO) Queuing System

$$P_n=\begin{cases}\dfrac{1}{n!}\rho^{n}P_0 & ;1\le n\le C\\ \dfrac{1}{C^{n-C}C!}\rho^{n}P_0 & ;n>C\end{cases}$$

$$E(m)=\frac{\lambda\mu\left(\dfrac{\lambda}{\mu}\right)^{C}P_o}{(C-1)!(C\mu-\lambda)^{2}}\qquad E(n)=E(m)+\frac{\lambda}{\mu}$$

$$P_0=\left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^{n}+\frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^{C}\frac{C\mu}{C\mu-\lambda}\right]^{-1}$$

$$E(w)=\frac{1}{\lambda}E(m)\qquad E(v)=E(w)+\frac{1}{\mu}$$

(M/M/C): (N/FIFO) Model

$$P_n=\begin{cases}\dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^{n}P_0 & ;0\le n\le C\\ \dfrac{1}{C^{n-1}C!}\left(\dfrac{\lambda}{\mu}\right)^{n}P_0 & ;C<n\le N\end{cases}$$

$$P_0=\begin{cases}\left[\displaystyle\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^{n}+\frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^{C}\left\{1-\left(\frac{\lambda}{C\mu}\right)^{N-C+1}\right\}\frac{C\mu}{C\mu-1}\right]^{-1} & ;\dfrac{\lambda}{C\mu}\neq 1\\ \left[\displaystyle\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^{n}+\frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^{C}(N-C+1)\right]^{-1} & ;\dfrac{\lambda}{C\mu}=1\end{cases}$$

$$E(m)=\frac{P_o(C\rho)^{C}\rho}{C!(1-\rho)^{2}}\left[1-\rho^{N-C+1}-(1-\rho)(N-C+1)\rho^{N-C}\right]\qquad E(w)=E(v)-\frac{1}{\mu}$$

$$E(n)=E(m)+C-P_0\sum_{n=0}^{C-1}\frac{(C-n)(\rho C)^{n}}{n!}\qquad E(v)=\left[E(n)\right]\Big/\lambda',\quad \text{where } \lambda'=\lambda(1-P_N)$$

(M/M/R):(K/GD) Model

$$P_n=\begin{cases}\dbinom{K}{n}\left(\dfrac{\lambda}{\mu}\right)^{n}P_0 & ;\ 0\le n<R\\ \dbinom{K}{n}\dfrac{n!}{R^{n-R}R!}\left(\dfrac{\lambda}{\mu}\right)^{n}P_0 & ;\ R\le n\le K\end{cases}\qquad E(m)=\sum_{n=R}^{K}(n-R)P_n$$

$$P_0=\left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^{n}+\sum_{n=R}^{K}\binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^{n}\right]^{-1}\qquad E(w)=\frac{E(m)}{\lambda\left[K-E(n)\right]}$$

$$E(n)=P_0\left[\sum_{n=0}^{R-1}n\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^{n}+\frac{1}{R!}\sum_{n-R}^{K}n.\binom{K}{n}\frac{n!}{R^{n-R}}\left(\frac{\lambda}{\mu}\right)^{n}\right]\qquad E(v)=\frac{E(n)}{\lambda\left[K-E(n)\right]}$$