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The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 05
Open Book Test (OBT) – 2017/2018
Applied Mathematics
ADU5303– Newtonian Mechanics II
Duration :- One Hour

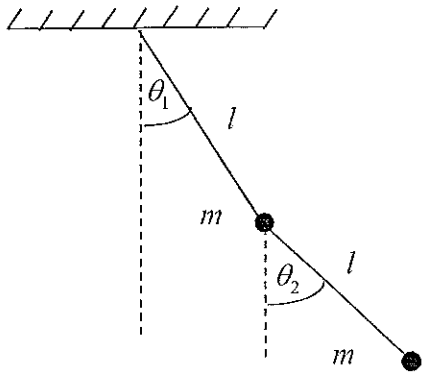
Date :- 13-01-2019 **Time :- 10.30 a.m. – 11.30 a.m.**

Answer All Questions.

1.(a) A uniform rod OA of length a, free to turn about its end O, revolves with uniform angular velocity ω about the vertical OZ through O, and is inclined at a constant angle α to OZ. Using D'Alembert principle, show that the value of α is either zero or $\cos^{-1}\left(3g/2a\omega^2\right)$.

(b) A rod, of length $2a$, revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semi-vertical angle α. Show that $\omega^2 = 3g/(2a\cos\alpha)$. Prove also that the direction of reaction at the hinge makes with the vertical an angle $\tan^{-1}\{(3/4)\tan\alpha\}$.

2. The double pendulum consists of two bobs of masses m at ends of two weightless rods of length l and one of them is fixed to a rigid support as shown in figure.



(a) Show that the kinetic energy is given by

$$T = ml^2\dot{\theta}_1^2 + \frac{1}{2}ml^2\dot{\theta}_2^2 + ml^2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2)$$

(b) Show that the potential energy is given by

$$V = -2mgl\cos\theta_1 - mgl\cos\theta_2$$

Hence obtain the Lagrangian of the system.

(c) Show that the Lagrange's equations of motion can be written as

$$2l\ddot{\theta}_1 + l\ddot{\theta}_2 \cos(\theta_2 - \theta_1) - l\dot{\theta}_2^2 \sin(\theta_2 - \theta_1) + 2g \sin\theta_1 = 0 \text{ and}$$

$$l\ddot{\theta}_2 + l\ddot{\theta}_1 \cos(\theta_2 - \theta_1) + l\dot{\theta}_1^2 \sin(\theta_2 - \theta_1) + g \sin\theta_2 = 0.$$