

THE OPEN UNIVERSITY OF SRI LANKA
 BACHELOR OF TECHNOLOGY/ BACHELOR OF
 SOFTWARE ENGINEERING – LEVEL 05
 FINAL EXAMINATION – 2014/2015
 MPZ5140/ MPZ5160 – DISCRETE MATHEMATICS II
 DURATION: THREE (03) HOURS.



Date: 03rd August 2015

TIME: 0930hrs – 1230 hrs.

Instruction:

- Answer only six questions.
- Please answer a total of six questions choosing at least one from each single section.
- State any assumption that you made.
- All symbols are in standard notation.

SECTION – A

01. i. Let A be any nonempty set with the operation " * " defined by $x * y = y$. Is the operation,
- a) associative? [10%]
- b) commutative? [10%]
- ii.
- a) Define a group on the set of real numbers with usual notation. [10%]
- b) Show that $\mathbb{R} - \{0\}$ is a group under ordinary multiplication operation. [30%]
- iii. Define a semi-group in usual notation. Let " * " be the operation on \mathbb{Q} defined by the following two ways:
- c) $x * y = 2(x + y)$
- d) $x * y = \frac{xy}{5}$
- Verify that whether $(\mathbb{Q}, *)$ is a semi- group for each of the above case. [40%]

02. i. Let $G = \{i, -i, 1, -1\}$ be a set with complex multiplication operation. Show that G forms abelian group under the operation " $*$ ". (note : $i^2 = -1$) [40%]
- ii. Let $G_1 = \left\{ \begin{pmatrix} p & q \\ 0 & r \end{pmatrix} \mid pr \neq 0, p, q, r \in \mathbb{Z} \right\}$. Show that $(G_1, *)$ is a group, where " $*$ " denotes the usual matrix multiplication. Verify whether G_1 is an abelian or not?. [60%]
03. i. Define a Homomorphism and Isomorphism for groups in usual notation. [20%]
- ii. Let $G = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \mid a \in \mathbb{R} \right\}$, and $G' = \mathbb{R}$. Assuming that G and G' are groups under the usual matrix multiplication.
- Let the mapping $\phi: G \rightarrow G'$, defined by $\phi(A) = a$ where $A = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$ for all $A \in G$.
- Show that ϕ is both Homomorphism and Isomorphism. [30%]
- iii. Let G be a group. Show that every element a in G has exactly one inverse in G . [20%]
- iv. If $\phi(e) = e$ and if ϕ is a homomorphism of G into G' , then show that $\phi(x^{-1}) = [\phi(x)]^{-1} \forall x \in G$. [30%]

SECTION – B

04. i. By drawing each of the following graphs, indicate which are simple graphs or not.
- a) $G_1 = \{V_1, E_1\}$, where $V_1 = \{a, b, c, d, e, f\}$ and $E_1 = \{\{a, b\}, \{a, d\}, \{a, f\}, \{b, d\}, \{b, f\}, \{b, e\}, \{d, e\}, \{e, f\}\}$. [10%]
- b) $G_2 = \{V_2, E_2\}$, where $V_2 = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and two vertices " s " and " t " are adjacent if and only if $\gcd(s, t) = 1$ where $s, t \in V_2$. [20%]
- c) $G_3 = \{V_3, E_3\}$, where $V_3 = \{1, 2, 3, \dots, 9, 10\}$, such that " i " and " j " in $V_3(G_3)$ are adjacent if and only if $i + j$ is a multiple of 4 where $i, j \in V_3$. [20%]
- ii. Construct a multi-graph of 6 vertex and 7 edges in which every vertex is odd. [15%]

- iii. Briefly explain connected graph, and disconnected graphs. [10%]
- iv. Find the number of vertices of a complete graph which has at least 800 edges. [25%]
05. i. What is the largest possible number of vertices in a graph with 2160 edges if all vertices have degree at least three? [20%]
- ii. Let G be a graph of 16 vertices and 29 edges in which every vertex is of degree 3 or 8. How many vertices of degree 3 and 8 does G have? Construct one such graph G . [40%]
- iii. Let $G(V, E)$ be a connected graph with at least two vertices and suppose that $|E(G)| < |V(G)|$. Prove that G has at least one vertex of degree one. [40%]
06. i. Define a "Tree graph" and draw a tree with 10 vertices, exactly 6 of which have degree 1 and one of which have degree 4. [20%]
- ii. G is the graph whose adjacency matrix P is given by
- $$P = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$
- a) Let $V(G) = \{p_1, p_2, p_3, p_4\}$. Find the number of paths of length four (4) joining vertices p_2 and p_4 . Give the list of paths if any. [50%]
- b) Without drawing a diagram of G , determine whether G is connected or not. [20%]
- a) Draw the graph of adjacency matrix P . [10%]

SECTION – C

07. i. Iterate the Eco-system growth model for the relation $k_{n+1} - \lambda k_n = 0$ for $k_0 = 0.3$, taking $\lambda = 0.3$ and $\lambda = 1.3$, and draw the diagram. Hence deduce k_n as $n \rightarrow \infty$. [40%]
- ii. Iterate the Eco-system growth model relationship $y_{n+1} - \lambda y_n - \lambda y_n^2 = 0$, where $\lambda = 2.6$ and $y_0 = 0.3$ (at least 6 iteration steps are necessary) and draw the graph for the relation. [40%]

- iii. Explaining the results of (i) and (ii), suggest an appropriate model for the population growth Eco-system. [20%]

08. A three dimensional system is governed by the following system of differential equations:

$$\frac{dx}{dt} = -9x - 3y - 7z,$$

$$\frac{dy}{dt} = 3x + y + 3z,$$

$$\frac{dz}{dt} = 11x + 3y + 9z.$$

Where $x, y,$ and z are function of t and at $t = 0, (x, y, z) = (1, 1, 0).$

Find the phase space value $(x(t), y(t), z(t))$ for $t = 1, 2.$ [100%]

09. i. Let $L_1 = \{a, ab, c\}$ and $L_2 = \{a^2, ab, b^2\}$ be two languages. Find $L_1L_2, L_2L_1,$ and $L_1^2.$ [15%]

- ii. Define the term "Grammar".

a) Let $G = \{S, \{0, 1\}, P, S\}$ and $P = \{S \rightarrow 11S, S \rightarrow 0\}.$ Find $L(G).$

b) Construct the production rule to generate each of the following language:

$$L_3 = \{a^l b^m c^n : l, m, n \geq 1\} \text{ and } L_4 = \{(ab)^n : n \geq 1\}. \quad [40\%]$$

- iii. Let M be a finite-state machine with state table given below: [45%]

| State | Input | | | Output | | |
|-------|-------|-------|-------|--------|-----|-----|
| | a | b | c | a | b | c |
| S_0 | S_0 | S_3 | S_2 | 0 | 1 | 1 |
| S_1 | S_1 | S_1 | S_3 | 0 | 0 | 1 |
| S_2 | S_1 | S_2 | S_3 | 1 | 1 | 0 |
| S_3 | S_2 | S_3 | S_0 | 1 | 0 | 1 |

- a) Find the input set, the state set, and the output set.
 b) Draw the state diagram of $M.$
 c) Starting at $S_0,$ what is the output for the input string $abbccc?$

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