THE OPEN UNIVERSITY OF SRI LANKA
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
BACHELOR OF SOFTWARE ENGINEERING

ECZ3161 – MATHEMATICS FOR COMPUTING
FINAL EXAMINATION – 2014/15 **CLOSED BOOK**

Date: August 1, 2015 **Time: 09.30-12.30 Hrs**

Instructions

1. Answer any **five** out of eight questions. All question carry equal marks.
2. Show all steps clearly.
3. **Programmable** calculators are **not** allowed.

Q1

(a) Use Boolean algebra to verify the following equations. **(4x3 marks)**

i) $\bar{a}\bar{b}c + a\bar{b}c + \bar{a}bc + abc = \overline{\bar{a}\bar{b}c + a\bar{b}c + \bar{a}bc + abc}$

ii) $\overline{\bar{a}\bar{b}cd + \bar{a}b\bar{c}d + abcd + ab\bar{c}\bar{d}} = \bar{a}c + a\bar{c} + \bar{b}d + b\bar{d}$

iii) $\overline{\bar{a}\bar{b}cd + abcd + a\bar{b}cd + abc\bar{d}} = \bar{c} + \bar{a}b + \bar{b}\bar{d} + \bar{a}\bar{d}$

(b) Use Truth tables to show the followings. **(4x2 marks)**

i) $xy + x\bar{z} = xyz + xy\bar{z} + x\bar{y}\bar{z}$

ii) $\overline{\bar{x} + y} + \bar{x}y = x\bar{y} + \bar{x}y$

Q2

(a)

If $A = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$, show that

$A^2 = 3A - 2I$; where I is the identity matrix of order 2. **(6 marks)**

(b)

Let $A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$, show that $A^2 = A$

Hence deduce that $(I - A)^2 = (I - A)$, where I is the identity matrix of order 3

(8 marks)

(c)

Let $A = \begin{pmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{pmatrix}$

Show that $A^2 = I$, where I is the identity matrix of order 3. **(6 marks)**

Q3 Consider 3×3 matrix A,

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$$

(a) Find AA^T **(6 marks)**

(b) Find the inverse of the matrix A using elementary raw operations. **(14 marks)**

Q4

(a) Given that $\sin\theta = \frac{3}{8}$, θ in quadrant I, and $\sin\alpha = \frac{5}{13}$, α in quadrant 2. Find

i) $\sin(\theta - \alpha)$ ii) $\cos(\theta + \alpha)$ iii) $\tan(\theta - \alpha)$

Give exact answers and show all your work. **(2x3 marks)**

(b) Sketch the graph of $y = 2\sin^2 x$ in the period $0 \le x \le 2\pi$. **(6 marks)**

(c) Answer the following problems.

i) At a point on a horizontal plane the elevation of a mountain is $22°15'$ and at another point on the plane 1 km farther away in a direct line, its elevation is $10°12'$.
Find the height of the mountain. **(4 marks)**

ii) The angle of elevation of the top of an unfinished tower at a point distance 120 m from its base is $30°$. How much higher must the tower be raised so that its angle of elevation at the same point may be $60°$? **(4 marks)**

Q5

(a) Given that $A + B = \frac{\pi}{2}$ and $\tan A = \frac{n}{n+1}$.

Show that, $\tan B = \frac{n+1}{n}$ and $\tan(A - B) = \frac{-2n-1}{2n(n+1)}$ **(4 marks)**

(b) Prove the following. **(4x3 marks)**

i) $\dfrac{\sin 2A+\sin 2B}{\sin 2A-\sin 2B}=\dfrac{\tan(A+B)}{\tan(A-B)}$

ii) $\dfrac{1+\cos x+\sin x}{1-\cos x+\sin x}=\dfrac{1+\cos x}{\sin x}$

iii) $\sin 50^\circ-\sin 70^\circ+\sin 10^\circ=0$

(c) If $A=36^\circ$, show that $\sin 3A=\sin 2A$.

Deduce that $\cos 36^\circ=\dfrac{\sqrt{5}+1}{4}$ **(4 marks)**

Q6

(a) Find the following limits **(4x3 marks)**

i) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$

ii) $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt{3+x^2}-2}$

iii) $\lim\limits_{x\to 4}\dfrac{x-4}{x^2-3x-4}$

(b) i) Taking $x_0=1$ as an initial approximation for the root of Newton-Raphson formula, obtain two further approximations to the positive root of $x^2-3=0$, giving your answers to 3 decimal places. **(4 marks)**

ii) By using Newton-Raphson method, find the root of $x^4-x-10=0$, which is near to $x=2$ correct to three places of decimal. **(4 marks)**

Q7

(a) Find first derivatives of the following from first principles. Show all steps. **(4x2 marks)**

i) x^2+5x-1 ii) $\cos x$

(b) Find $\dfrac{dy}{dx}$ of, **(3x2 marks)**

i) $y=\dfrac{x-1}{\sqrt{x}}$ ii) $y=2\cos^2\left(x^2+1\right)$

(c) If $y=\dfrac{1-3x^2}{1+3x^2}$ show that

$\left(1+3x^2\right)^2\dfrac{dy}{dx}+12x=0$ **(6 marks)**

Q8

(a) Evaluate the following. **(3x2 marks)**

i) $\int (2\cos x - 3\sin x)dx$ ii) $\int \sin(3x-1)dx$

(b) Find the exact value of the following. **(4x2 marks)**

i) $\int_0^2 (x^2 - x + 1)dx$ ii) $\int_0^{\frac{\pi}{2}} (\sin x - \cos x)dx$

(c) i) Sketch the graphs of the parabola $y = x^2$ and the straight line $y = x$ in the same figure.
ii) Find the coordinates of the intersection of the curves in above.
iii) Hence, find the area bounded by the two curves, $y = x^2$ and $y = x$.

(2x3 marks)

END.