

THE OPEN UNIVERSITY OF SRI LANKA
 BACHELOR OF TECHNOLOGY HONORS IN ENGINEERING /
 BACHELOR OF SOFTWARE ENGINEERING HONORS – LEVEL 05
 FINAL EXAMINATION – 2015/2016



MPZ5140/ MPZ5160 – DISCRETE MATHEMATICS II

DURATION: THREE (03) HOURS.

Date: 16th November 2016

TIME: 1330hrs – 1630 hrs.

Instruction:

- Answer only six questions.
- Please answer a total of six questions choosing at least one from each single section.
- All symbols are in standard notation and state any assumption that you made.

SECTION – A

01. i. Let $(S, *)$ be an algebraic system such that $(a * b) * a = a$ for all $a, b \in S$.
 Show that $a * (a * b) = a * b$ for all $a, b \in S$. [10%]
- ii. Define a semi-group in usual notation.
 Let " $*$ " be a binary operation on \mathbb{Z} defined by the following two ways:
- a) $x * y = x - y$ for all $x, y \in \mathbb{Z}$.
 b) $a * b = a(1 + b) + b(1 + a)$ for all $a, b \in \mathbb{Z}$.
 Verify that whether $(\mathbb{Z}, *)$ is a semi- group for each of the above case. [50%]
- iii. Let " $*$ " be a binary operation defined on $S = \mathbb{Q} \times \mathbb{Q}$, the set of all ordered pairs of rational numbers, by $(a, b) * (x, y) = (ax, ay + b)$ for all $(a, b), (x, y) \in S$.
 Show that $(S, *)$ is a semi-group or not. [40%]
02. i. Define a group and an Abelian group with usual notation. [15%]
- ii. Prove that \mathbb{Z} is an abelian group with respect to the binary operation " $*$ ".
 Defined by $x * y = x + y + 5xy$, for all $x, y \in \mathbb{Z}$. [60%]
- iii Consider the group $(G_1, *)$, where $G_1 = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \mid a^2 + b^2 \neq 0, a, b \in \mathbb{R} \right\}$.
 and " $*$ " denotes the usual matrix multiplication. Show that $(G_1, *)$ is an Abelian group. [25%]

03. i. Define a Homomorphism and Isomorphism for groups in usual notation. [30%]
- ii. Let $(\mathbb{Z}, +)$ and $(\mathbb{Z}', +)$ be the groups of integers and even integers. Define a map $f: \mathbb{Z} \rightarrow \mathbb{Z}'$ such that $f(x) = 2x$ all $x \in \mathbb{Z}$.
Show that f is an Isomorphism.
[40%]
- iii. Show that the mapping ϕ from the symmetric group p_n onto the multiplicative group $G_2 = \{-1, 1\}$, defined by $\phi(\alpha) = 1$ or -1 , according as α is even or odd permutation in p_n , is a homomorphism of p_n onto G_2 . [30%]

SECTION – B

04. i. By drawing each of the following graphs, determine which of the following graphs are simple.
- a) $G_1 = \{V_1, E_1\}$ where $V_1 = \{1, 2, 3, \dots, 9, 10\}$ and
 $E_1 = \{\{x, y\} \mid x + 2y \text{ is odd and } x < y\}$ [15%]
- b) $G_2 = \{V_2, E_2\}$, where $V_2 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and
 $E_2 = \{\{i, j\} \mid i \times j \text{ is a divided by 5 and } i \neq j\}$ [15%]
- c) $G_3 = \{V_3, E_3\}$, where $V_3 = \{v, v_2, v_3, v_4, v_5, v_6, v_7\}$ and
 $E_3 = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6\}$ with $\rho_1 = (v_1, v_4)$, $\rho_2 = (v_1, v_2)$,
 $\rho_3 = (v_2, v_3)$, $\rho_4 = (v_4, v_3)$, $\rho_5 = (v_5, v_7)$, and $\rho_6 = (v_2, v_5)$. [15%]
- ii. Draw all possible simple, connected graphs with 6 vertices and 15 edges. [10%]
- iii. Find the number of vertices of a complete graph which has at least 1000 edges. [25%]
- iv. Show that there is no simple graph with 12 vertices and 28 edges in which
a) the degree of each vertex is either 3 or 4
b) the degree of each vertex is either 3 or 6. [20%]
05. i. Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 2. Draw two such graphs. [30%]
- ii. For each of the following degree sequences given bellow, find a graph, if it exists. In the case of exist, draw a corresponding graph. Otherwise, give reasons for the non-existence.
- a) 4, 4, 4, 3, 2 b) 3, 3, 3, 3, 2 c) 5, 4, 3, 2, 1, 1. [30%]

- iii. Prove that a complete graph with n vertices contains $n(n - 1)/2$ edges. [40%]
06. i. Draw the trees for the following cases:
- a) twelve vertices, at least half of which has degree one, [15%]
 b) six vertices exactly three of which has degree one. [15%]

- ii. Let G be a graph whose adjacency matrix A is given by

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

- a) Let $V(G) = \{a_1, a_2, a_3, a_4\}$. Find the number of paths of length four (4) joining vertices a_3 and a_1 . Give the list of paths if they exist. [40%]
- b) Without drawing a diagram for G , determine whether G is connected or not. [20%]
- c) Draw the graph corresponding to the adjacency matrix A . [10%]

SECTION - C

07. i. For the given relationship $x_{n+1} = \lambda x_n(1 - x_n)$, where $\lambda = 1.6$, obtain the convergent value in the for 4 digits each of the following case:
- a) $x_0 = 0.3$ b) $x_0 = 0.6$ [40%]
- ii. Iterate the Eco-system growth model relationship $t_{n+1} = \lambda t_n(1 - t_n)$, where $\lambda = 2.1$ and $t_0 = 0.3$ (at least 6 iteration steps are necessary) and draw the graph for the relation. [20%]
- iii. Iterate the relation $Z_{n+1} = Z_n^2 + \lambda$, where $\lambda = 0$ (at least 5 iteration steps are necessary), and draw the diagram for each of the following case:
- a) $Z_0 = 2.2 + i0.5$ b) $Z_0 = 0.3 + i0.5$
- Hence, find Z_n as $n \rightarrow \infty$ for each case. [40%]

08. A three dimensional system is governed by the following system of differential equations:

$$\frac{dy_1}{dt} = y_1 + 2y_2 - y_3$$

$$\frac{dy_2}{dt} = y_1 + y_3$$

$$\frac{dy_3}{dt} = 4y_1 - 4y_2 + 5y_3,$$

where $y_1, y_2,$ and y_3 are function of t and at $t = 0, (y_1, y_2, y_3) = (1, 1, 0)$.

Find the phase space value $\{y_1(t), y_2(t), y_3(t)\}$ for $t = 1, 2$.

[100%]

09. i. Consider the grammar G with $V = \{S, A, B\}, \Sigma = \{a, b\}$ and $P = \{S \rightarrow aABa, A \rightarrow baABb, B \rightarrow Aab, aA \rightarrow baa\}$ in the usual notation. Verify whether the string $w = baabbabaaababa \in L(G)$. [25%]
- ii. Consider the language $L_1 = \{1, 2, 12\}$ and $L_2 = \{a, b, ab\}$. Find L_1L_2 , and L_2^3 .
- iii. Let M be a Mealy machine with the following functions. Let $s \in S, a \in I$ and $x \in I^*$. [30%]

$\delta: S \times I^* \rightarrow S$ and $\beta^*: S \times I^* \rightarrow O^*$ by

$$\delta^*(s, \Omega) = s,$$

$$\delta^*(s, a.x') = \delta^*[\delta(s, a), x'],$$

$$\beta^*(s, \Omega) = \Omega,$$

$$\beta^*(s, a.x') = \beta(s, a)\beta^*[\delta(s, a), x'].$$

The two frame binary pipeline device hold up two binary as in the following table.

State	Input		Output	
	0	1	0	1
000	010	001	0	1
001	101	001	1	0
010	011	100	0	0
011	010	011	0	1
100	100	111	1	1
101	101	110	1	0
110	110	101	1	1
111	111	101	0	1

Find the two frame binary pipeline buffer and work out its response to the sequence 011011 from the state 100.

[45%]

- Copyrights reserved -