THE OPENUNIVERSITY OF SRI LANKA
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
BACHELOR OF SOFTWARE ENGINEERING

ECZ3161 – MATHEMATICS FOR COMPUTING
FINAL EXAMINATION – 2015/16 **CLOSED BOOK**

Date: 04 December 2016 **Time: 09.30-12.30Hrs**

Instructions

1. Answer any **five** out of eight questions. All question carry equal marks.
2. Show all steps clearly.
3. **Programmable** calculators are **not** allowed.
4. Total marks obtaining for this examination is 100. The marks assigned for each question is in square brackets.

Q1

(a) Use Boolean algebra to prove the following equations. **[6]**

i) $AB + \overline{A}C + BC = AB + \overline{A}C$

ii) $(\overline{AB})(\overline{A} + B)(\overline{B} + B) = \overline{A}$

iii) $\overline{A\overline{B} + B\left(\overline{C} + \overline{CD}\right)} = \overline{AB} + \overline{A}CD + BCD$

(b) Find which of the following propositions are tautologies, contradictions or neither. **[4]**

i) $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$

ii) $(P \wedge Q) \wedge \neg(P \vee Q)$

(c) Use Karnaugh maps and find minimal sum for followings. **[10]**

i) $\overline{A}B + B\overline{C} + BC + \overline{ABC}$

ii) $\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$

Q2

(a) Given that A = $\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 3 & 4 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}$, show that $(A\ B\ )^T = B^T A^T$ **[6]**

(b) If A=$\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$, **[6]**

i) show that $A^3 = I$

ii) Hence, find A^{-1}

(c) Use Gauss-Jordan elimination to solve the linear system. **[8]**

$-x + 2y + 6z = 2$
$3x + 2y + 6z = 6$
$x + 4y - 3z = 1$

Q3

(a)

i) Find all of the angles between 0° and 360° (or between 0 and 2π radians) that satisfy the following conditional relationship. **[2]**
$\sin x - 1 = \cos x$

ii) Find the exact value of $\sin(52.5^0)\cos(7.5^0)$. **[2]**

iii) Given that $\sin(\theta) = 3/5$ and that $\cos(\theta) = 4/5$ find $\sin(2\theta)$ and $\cos(2\theta)$. **[2]**

(b) On the same set of axes from 0 *to* 2π, plot, **[6]**

$$y = 2\cos(\frac{1}{2}x) \quad \text{and} \quad y = \sin(2x)$$

(c)

I) A tree stands at the top of a 5m mountain. From a point from the ground, the angle elevation of the top of the tree is 60^0 and from the same point, the angle elevation of the top of the mountain is 45^0. Find the height of the tree. **[4]**

II) From a point A, a man observes that the angle of elevation of the summit of a hill is 30°. He then walks towards the hill for 500 m along flat ground. The summit of the hill is now at an angle of elevation of 45°. Draw a diagram and find the height of the hill above the level of A to the nearest metre. **[4]**

Q4

(a) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 5 & 0 \end{bmatrix}$ find matrix **X** from **X+A+B=0.** **[6]**

(b)

I) Show that $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is an orthogonal matrix. **[4]**

II) Show that the matrix $\begin{bmatrix} i & -3+4i & 2+i \\ 3+4i & 0 & -1-i \\ -2+i & 1-i & 4i \end{bmatrix}$ is a skew-hermitian matrix.

[4]

(c) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$, show that $A^3 - 3A^2 - I = 0$ where I is the identity matrix of order 3. **[6]**

Q5

(a) Prove the following trigonometric identities. **[6]**

i) $4\sin^4\theta + \sin^2 2\theta + 2\cos 2\theta = 2$

ii) $\tan 3\theta \cot\theta = \dfrac{(1+2\cos 2\theta)}{(2\cos 2\theta - 1)}$

iii) $\sin^2\left(\dfrac{\pi}{4}+\theta\right) - \sin^2\left(\dfrac{\pi}{4}-\theta\right) = \sin 2\theta$

(b) Solve $\mathbf{\sin\theta + \sin 3\theta + \sin 5\theta = 0}$ within the rage $0<\theta<\pi$. **[6]**

(c)

i) If $A+B+C=\pi$ prove that $\sin(A+B)+\sin(B+C) = 2\cos\dfrac{B}{2}\cos\left(\dfrac{A-C}{2}\right)$. **[4]**

ii) Prove that $\sin^{-1}\dfrac{3}{5} + \sin^{-1}\dfrac{8}{17} = \sin^{-1}\dfrac{77}{85}$ **[4]**

Q6

(a) Find the following limits. **[8]**

i) $\lim\limits_{x\to\infty}\left(\dfrac{2x^2+3x-2}{3x^2-x+1}\right)$

ii) $\lim\limits_{x\to 0}\left(\dfrac{2\sin x - \sin 2x}{x^3}\right)$

iii) $\lim\limits_{x\to 0}\left(\dfrac{x+\sin x}{x-\sin 3x}\right)$

iv) $\lim\limits_{x\to 10}\left(\dfrac{\sqrt{x-1}-2}{x^2-25}\right)$

(b) Differentiate the following functions with respect to x. **[6]**

i) $e^{-2x}\sin^{2x}$

ii) $\left(\dfrac{x\sin x}{1+\cos x}\right)$

(c) Find the equation of the tangent to the curve $y=\sin(3x)+1$ at the point where $x=\dfrac{\pi}{3}$. **[6]**

Q7

(a) Find first derivatives of the following from first principles. Show all steps. **[6]**

i) $x^2 + 2x + 3$

ii) $x^2 \sin x$

(b) In a laboratory, you are given a block of aluminum. You measure the dimensions of the block and its displacement in a container of a known volume of water. You calculate the density of the block of aluminum to be 2.68 g/cm^3. You look up the density of block aluminum at room temperature and find it to be 2.70 g/cm^3. Calculate the percent error of your measurement. **[4]**

(c)

i) For the following table of values, estimate f (7.5) using Newton's backward interpolation formula. **[5]**

x	1	2	3	4	5	6	7	8
y=f(x)	1	8	27	64	125	216	343	512

ii) Use Newton-Raphson method to find a root near 2, of the following equation.

$x^3 - 2x - 5 = 0$. **[5]**

Q8

(a) Evaluate the following. **[6]**

i) $\int x(x^2 - 2)^4 dx$

ii) $\int \sin^2 2x \cos^2 2x dx$

iii) $\int \sqrt{1 + \cos x} dx$

(b) Find the exact value of the followings. **[6]**

i) $\int_0^{\frac{\pi}{2}} x \sin^2 x dx$

ii) $\int_0^1 x^3 \ln x dx$

(c) Given that $I = \int_0^{\frac{\pi}{2}} e^x \sin x dx$ and $J = \int_0^{\frac{\pi}{2}} e^x \cos x dx$ .Show that $I - J = 1$. Find the values of I and J. **[8]**

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