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THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering /
Bachelor of Software Engineering Honors

Final Examination (2016/2017)
MPZ5140 /MPZ5160: Discrete Mathematics II

Date: 08th November 2017 (Wednesday) **Time: 13:30 pm – 16:30 pm**

Instruction:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

SECTION – A

Q1.

I. Define a semi-group in usual notation.

Let "#" be the operation on $\mathbb{R}$ defined by the following ways:

a) "$a \# b = ab + 1$ for all $a, b \in \mathbb{R}$;
b) "$x \# y = \sqrt{xy}$" for all $x, y \in \mathbb{R}$;
c) "$l \# m = \frac{3}{5}\, lm$" for all $l, m \in \mathbb{R}$.

Verify that where $(\mathbb{R}, \#)$ a semi-group is for each of the above case. [40%]

II. Let $R = \{0, 1, 2, 3, 4, 5\}$ be a group under the operation $\oplus_6$. The operation $\oplus_6$ is defined by $a \oplus_6 b = r$ and $0 \leq r \leq 5$, where r is the non-negative remainder when ordinary addition $a + b$ is divided by 6. [35%]

a) Determine the identity element of R.
b) Determine the inverse of each element $a \in R$.

III. Let the operation "$*$" is defined on the set of real number $\mathbb{R}$ as follows:

$$a * b = |a - b| \quad \text{for all } a, b \in \mathbb{R}.$$

Prove that "$*$" is the commutative binary operation on $\mathbb{R}$ but $(\mathbb{R}, *)$ is not a semi group. [25%]

Q2.

I. Define a group and an Abelian group with usual notation [15%]

II. Let $A = \left\{r - \frac{1}{3} \mid r \in \mathbb{Q}\right\}$. The binary operation " $*$ " on A is defined by

$$m * n = m + n + 3mn \quad \text{for all } m, n \in A.$$

Prove that A is a group. [55%]

III. Let $G_\times$ denotes the group of all 2×2 square matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$ under the matrix multiplication, where all $a \in \mathbb{R}^+$.
G_+ denoted the group of all 2×2 square matrices of the form $\begin{pmatrix} a & a \\ a & a \end{pmatrix}$ under the matrix addition, where all $a \in \mathbb{R}$ Determine where the groups $G_\times$ and G_+ are Abelian or non-Abelin. [30%]

Q3.

I. Define a Homomorphism and Isomorphism for group in usual notation. [20%]

II. Consider the group $(\mathbb{Z}, +)$. Define G by $\left\{\begin{pmatrix} x & -x \\ -x & x \end{pmatrix} \mid x \in \mathbb{Z}\right\}$. Assuming that $(G, +)$ is a group, show that $f: \mathbb{Z} \to G$ define by $f(x) = \begin{pmatrix} x & -x \\ -x & x \end{pmatrix}$ for all $x \in \mathbb{Z}$ is an isomorphism [30%]

III. For a fixed element a in a group G, define $f_a: G \to G$ by $f_a(x) = a^{-1}x\, a,$ for all $x \in G$. Show that f_a is a homomorphism. [20%]

IV. Let $H = \mathbb{Z}$ be the group under the addition and $\overline{H}$ be the group $\overline{H} = \{1, -1\}$ under the multiplication.
Define $\Phi: H \to \overline{H}$ by

$$\Phi(n) = \begin{cases} 1, & n \text{ is even} \\ -1, & n \text{ is odd} \end{cases}.$$

Show that Φ is a homomorphism. [30%]

SECTION – B

Q4.

I. By drawing each of the following graph, determine whether which graphs are simple or not

a) $G_1 = \{V_1, E_1\}$, where $V_1 = \{1, 2, 3, \ldots, 9, 10\}$ and $E_1\{\{x, y\} \mid 2x + 3y \text{ is even and } x < y\}$. [15%]

b) $G_2 = \{V_2, E_2\}$, where $V_2 = \{1, 2, 3, \ldots, 9, 10\}$, such that two number "$i$" and "$j$" are adjacent if and only if $|i - j| \le 3$, and $i < j$. [15%]

c) $G_3 = \{V_3, E_3\}$, where $V_3 = \{2, 3, 4, 5, 11, 12, 13, 14\}$ and two vertices $'s'$ and $'t'$ are adjacent if and only if $\gcd(s, t) = 1$, and $s < t$. [15%]

II. Define the terms "connected" and "complete" graph and draw the complete graph on seven vertices. [15%]

III. Find the number of vertices of a complete graph which has at least 800 edges [20%]

IV. Construct a graph for each of the following case: [20%]

a) multiple graph of six vertices and seven edges,
b) simple graph of seven vertices and nine edges.

Q5.

I. Let G be a graph of 12 vertices and 28 edges in which every vertex is of degree 4 or 6. How many vertices of degree 4 and 6 does G have? Construct one such graph G. [30%]

II. Is there a graph with degree 1, 3, 3, 5, 5, 7, 8, 8, 9 on nine vertices? Justify your answer. [20%]

III. Let H be a graph of 12 vertices and 17 edges, and x, y, z denoted by the number of vertices in H of degree 2, 3, and 4 respectively. Assume that $y \geq 3$. Find all possible answers for (x, y, z). For each your answer, construct a corresponding graph. [50%]

Q6.

I. G is the graph whose adjacency matrix P is given by

$$P = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

a) Let $V(G) = \{p_1, p_2, p_3, p_4\}$. Find the number of paths of length **four** joining vertices p_2 and p_3. Give the list of paths if any. [40%]
b) Without drawing a diagram of G, determine whether G is connected or not. [20%]
c) Draw the graph corresponding to the of adjacency matrix P. [10%]

II. Is it possible to draw the each of the following case?:

a) a tree with 11 vertices, each of which has either degree 1 or degree 3.
b) a tree with 22 vertices, each of which has either degree 1 or degree 3.

If it is possible, draw a tree graph. [30%]

SECTION – C

Q7.

I. Iterate the Eco-system grow model for the relation $k_{n+1} - \lambda k_n = 0$ with $k_0 = 0.4$, for all the cases $\lambda = 0.5$ and $\lambda = 1.2$, and draw the corresponding diagram for each λ. [30%]

II. For the given relationship $p_{n+1} = \lambda p_n(1 - p_n)$, where $\lambda = 1.8$, obtain the convergent value, and draw the diagram for each of the following case: [40%]

a) $p_0 = 0.4$,
b) $p_0 = 0.7$.

III. Explain mathematically as to why used the equation $p_{n+1} = \lambda p_n(1 - p_n)$ to represent the population growth in a limited eco-system, instead of the equation $p_{n+1} = \lambda p_n$. [30%]

Q8.

A three dimensional system is governed by the following system of differential equations:

$$\frac{dx_1}{dt} - 3x_1 - x_2 + 2x_3 = 0,$$

$$\frac{dx_2}{dt} + x_1 - 2x_2 - x_3 = 0,$$

$$\frac{dx_3}{dt} - 4x_1 - x_2 + 3x_3 = 0,$$

where x_1, x_2, and x_3 are function of t and at $t = 0, (x_1, x_2, x_3) = (1, 0, 4)$.

Find the phase space value $(x_1(t), x_2(t), x_3(t))$ for $t = 1, 2$. [100%]

Q9.

I. Consider the language $L_1 = \{5, 56, 6\}$ and $L_2 = \{65, 656, 565\}$. Find L_1L_2 and $L_1L_2^2$. [15%]

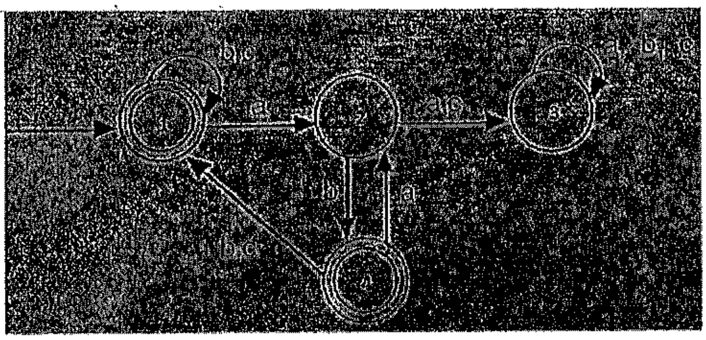
II. Construct the production rules to generate each of the following language: [25%]

a) $\{a^{2n} : n \geq 1\}$,

b) $\{(ab)^n : n \geq 1\} \cup \{(ba)^n : n \geq 1\}$.

III. Write down the regular languages (acceptor for strings) for following Finte-State Machines.

[10%]



IV. Draw the directed graph that describers the DFA (Deterministic Finite Automation) with the following state transition table.

State	Input			
	a	b	c	d
s_0	s_1	s_0	s_2	s_1
s_1	s_0	s_1	s_1	s_2
s_2	s_0	s_3	s_2	s_1
s_3	s_2	s_3	s_1	s_4
s_4	s_4	s_4	s_4	s_4

Initial state s_0 and accepting state s_4. [20%]

V. Let M be a Mealy machine. Let $s \in S, a \in I$ and $s \in I^*$ and defined functions

$\delta: S \times I^* \rightarrow S$ and $\beta^*: S \times I^* \rightarrow O^*$ by

$\delta^*(s, \Omega) = s,$

$\delta^*(s, a.x') = \delta^*[\delta(s, a), x'],$

$\beta^*(s, \Omega) = \Omega,$

$\beta^*(s, a.x') = \beta(s, a)\beta^*[\delta(s, a), x'].$

The two frame binary pipeline devise hold up two binary as in the following table,

State	Input		Output	
	0	1	0	1
00	01	00	1	0
01	01	10	0	1
10	11	00	0	0
11	01	00	1	1

Find the two frame binary pipeline buffer and work out its response to the sequence 1110110 from the state 10. [30%]